THE DIRICHLET PROBLEM FOR HOMOGENEOUS ELLIPTIC OPERATORS IN A HALF SPACE

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In a recent series of papers, Lions and Magenes [4], [5] study extensively boundary-value problems for elliptic operators. A prototype of problem that cannot be attacked by their methods is the Dirichlet problem for powers of the Laplacian in a half space. By using the completion of the space of smooth functions with respect to the Dirichlet norm (§1), we are able to solve this problem.

We obtain, then, for a general class of homogeneous elliptic operators defined in a half space, isomorphism theorems establishing existence and uniqueness for the Dirichlet problem (§§2, 4 and 5), a regularity result (§3) and trace theorems (§§2, 3 and 4). Using the theory of interpolation [3] we obtain other isomorphism theorems between general classes of interpolated spaces. Among these, we can characterize the spaces of boundary values (Theorem 5.2).

Proofs will appear elsewhere. The writer is indebted to J.-L. Lions for suggestions and criticism.

1. **Preliminaries.** Let R^n be the Euclidean space of n dimensions, its elements being denoted by $x=(x_1, \cdots, x_n)$. We denote by R^n_+ (resp. $\overline{R}^n_+$) the set of elements $x=(x_1, \cdots, x_n)\in R^n$ such that $x_n>0$ (resp. $x_n\geqq 0$). If $p=(p_1, \cdots, p_n)$ is an n-tuple of integers $\geqq 0$, let $D^p=D_1^{p_1}, \cdots, D_n^{p_n}$, where $D_j=(1/i)(\partial/\partial x_j)$, $1\leqq j\leqq n$, and let $|p|=p_1+\cdots+p_n$ be the order of D^p. If Ω is an open subset of R^n, we denote by $C_c^\infty(\Omega)$ the space of infinitely differentiable functions with compact support in Ω.

DEFINITION 1.1. *We denote by $\boldsymbol{D}^m(R^n)$ the completion of $C_c^\infty(R^n)$, with respect to the following norm*:

$$\|\phi\|_{\boldsymbol{D}^m(R^n)} = \left(\sum_{|p|=m} \|D^p\phi\|^2_{L^2(R^n)}\right)^{1/2}. \tag{1.1}$$

Clearly, $\boldsymbol{D}^m(R^n)$ is a Hilbert space. If $n>2m$ (an assumption that we shall make throughout this paper), we have the Sobolev inequality [6]:

$$\|\phi\|_{L^{q(m)}(R^n)} \leqq c\|\phi\|_{\boldsymbol{D}^m(R^n)}, \qquad \text{for all } \phi\in C_c^\infty(R^n), \tag{1.2}$$

where