LOCALLY FLAT STRINGS

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I. **The Schoenflies Theorem for strings.** In [1], Stallings defines a *string* of type (n, k) to be a pair (R^n, Y), where Y is a closed subset of R^n such that Y is homeomorphic to R^k. Similarly, he defines a pair (S^n, X), where X is homeomorphic to S^k, to be a *knot* of type (n, k). A pair (A, X) of (n, k)-manifolds is said to be *locally smooth* if each point of X has a neighborhood U in A such that the pair $(U, U \cap X)$ is homeomorphic to the pair (R^n, R^k). Thus, his definition of locally smooth is equivalent to Brown's [2] definition of locally flat.

Let (R^n, Y) be a locally smooth string of type $(n, n-1)$; Y separates R^n into two components whose closures are A and B. In [1], Stallings states that it seems possible that either A or B must be homeomorphic to a closed half-space of R^n. Harrold and Moise [3] have proved this for $n=3$. In this note we observe that both A and B are closed half-spaces of R^n for $n>3$ and hence we have a Schoenflies theorem for strings of type $(n, n-1)$ for $n>3$.

THEOREM I.1. *Let (R^n, Y) be a locally flat string of type $(n, n-1)$ and let A and B be the closures of the complementary domains of Y in R^n. Then A and B are homeomorphic to a closed half-space of R^n for $n>3$.*

COROLLARY I.2. *Let (R^n, Y) be a locally flat string of type $(n, n-1)$ for $n>3$. Then (R^n, Y) is trivial, that is, there is a homeomorphism h of (R^n, Y) onto $(R^n, R^{n-1}\times 0)$.*

COROLLARY I.3. *Let f_1, f_2 be two locally flat embeddings of R^{n-1} as a closed subset of R^n for $n>3$. Then there is a homeomorphism h of R^n onto R^n such that $hf_1=f_2$.*

Theorem I.1 follows immediately from a recent result of Cantrell's [4]. Cantrell showed that a knot (S^n, Y) of type $(n, n-1)$ is trivial for $n>3$ provided Y is locally flat except at one point. Thus, if (R^n, X) is a locally flat string of type $(n, n-1)$ and (S^n, Y) is the one point compactification of (R^n, X), Y is locally flat except at the compactification point. Hence (S^n, Y) is trivial for $n>3$ and Theorem I.1 follows.

II. **The Slab Conjecture.** In this section we consider the relationship of locally flat strings of type $(n, n-1)$ to the Annulus Conjecture. We now state the Annulus Conjecture.