

connecting the fixed points of  $T$ . (4) If  $O_1$  and  $O_2$  are disjoint simply connected domains invariant under a loxodromic  $T$ , the corresponding arcs, as in (3), divide  $S$  into two Jordan regions, one or the other of which must contain any domain disjoint from  $O_1$  and  $O_2$ . (5) If  $O$  is a simply connected domain invariant under an elliptic  $T$ , then  $O$  must contain a fixed point of  $T$ .

The examples are elaborations of the ideas in L. R. Ford, *Automorphic functions*, 2nd ed., Chelsea, 1951, pp. 55–59.

BROWN UNIVERSITY

---

## DIFFERENTIABLE NORMS IN BANACH SPACES<sup>1</sup>

BY GUILLERMO RESTREPO

Communicated by W. Rudin, January 20, 1964

**1. Introduction.** In [4, p. 28] S. Lang has asked whether or not a separable Banach space has an admissible norm of class  $C^1$ . In this note we indicate a proof of the following theorem, which characterizes those Banach spaces for which such a norm exists.

**THEOREM 1.** *A separable Banach space has an admissible norm of class  $C^1$  if and only if its dual is separable.*

It follows from this theorem that not even  $C(I)$  possesses an admissible differentiable norm.

**2. Preliminaries.** Let  $X$  be a Banach space with norm  $\alpha$ ; we write  $S_\alpha = \{x | \alpha(x) = 1\}$  and  $B_\alpha = \{x | \alpha(x) \leq 1\}$ . A norm in  $X$  is admissible if it induces the same topology as does  $\alpha$ . The dual space is written  $X^*$  and the norm dual to  $\alpha$  is denoted by  $\alpha^*$ . An  $f \in X^*$  is called a support functional to  $B_\alpha$  at  $x \in S_\alpha$  if  $\alpha^*(f) = f \cdot x$ ; if  $f$  has norm 1, it is called a normalized support functional and is written  $\nu_x$ . A norm is smooth if there is a unique normalized support functional to  $B_\alpha$  at each  $x \in S_\alpha$ . The norm  $\alpha$  is differentiable at  $x \neq 0$  if there is an  $\alpha'(x) \in X^*$  such that

$$\lim_{y \rightarrow x; y \neq x} \frac{|\alpha(y) - \alpha(x) - \alpha'(x) \cdot (y - x)|}{\alpha(y - x)} = 0$$

---

<sup>1</sup> Research partially supported by NSF Grant G-24471.