INVARIANT DOMAINS FOR KLEINIAN GROUPS[1]

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If the limit set, Σ, of a properly discontinuous group, Γ, of fractional linear transformations of the Riemann sphere, S, contains more than two points, call Γ Kleinian. Otherwise, call Γ elementary. Let $\{\Omega_i\}$ be an enumeration of the components of Ω, the set of discontinuity. If O is a domain in S, i.e., O is open and connected, let $\Gamma(O)$ be the subgroup of Γ of elements which map O onto itself. If $\Gamma(O)=\Gamma$, call O an *invariant domain.* If $\Gamma(\Omega_i)=\{id\}$, call Ω_i an *atom*.

THEOREM 1. *If Γ possesses three disjoint invariant domains then Γ is cyclic.*

THEOREM 2. *Suppose Γ possesses an invariant component Ω_0. If $0\neq i\neq j\neq 0$, then $\Gamma(\Omega_i)\cap\Gamma(\Omega_j)$ is a nonloxodromic and nonhyperbolic cyclic group. If Ω_0 is simply connected this latter group is nonelliptic.*

THEOREM 3. *If Γ is a Kleinian group with two disjoint invariant domains, then there exists a maximal pair of disjoint invariant domains[2] each of which is simply connected. All noninvariant components of Ω are atoms.*

The author is grateful to Leon Greenberg for pointing out how the next theorem follows from the methods used in proving the previous theorems and, essentially, from a deep theorem of Nielsen and Fenchel on Fuchsian groups.

THEOREM 4. *If O_1 and O_2 are a maximal pair of disjoint invariant domains for a Kleinian group, Γ, then O_1/Γ and O_2/Γ are homeomorphic surfaces.*

Examples are given where (a) Ω and Σ are both connected and (b) where Γ possesses two invariant components *and* atoms.

The proofs follow from remarks of which the following are typical. (1) A closed set, invariant under Γ, contains Σ. (2) The components of the complement of a closed connected set are simply connected. (3) If O is a simply connected domain invariant under a loxodromic transformation, T, then there is a Jordan arc in O invariant under T

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[2] *Added in proof.* O_1 and O_2 are a maximal pair of disjoint invariant domains if whenever O_1' and O_2' are a pair of disjoint invariant domains such that $O_i\subset O_i'$, then $O_i=O_i'$ for $i=1, 2$.