A THEOREM ON FILTERED LIE ALGEBRAS AND ITS APPLICATIONS

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The purpose of this note is to announce the following theorem and indicate a few of its applications to the classification of irreducible (complex or real) Lie algebras of linear endomorphisms of finite type of order 2 (or higher) as well as some questions on transformation groups acting on symmetric spaces. Other applications to homogeneous spaces as well as the proofs of all the results stated below will be published elsewhere.

THEOREM 1. *Let L be a Lie algebra of finite dimension over a field of characteristic zero and A a group of automorphisms of L satisfying the following conditions*:

(a) *There is a decreasing sequence of A-invariant subalgebras $L=L_{-1}\supset L_0\supset L_1\supset \cdots \supset L_k$ such that*

$$[L_p, L_q] \subset L_{p+q} \quad \textit{for } p, q = -1, 0, 1, \cdots, k,$$

where, by convention, $L_{-2}=L$ and $L_{k+1}=L_{k+2}= \cdots =0$.

(b) *For every $t\in L_p$, $p\geqq 0$, such that $t\notin L_{p+1}$, there is an element $x\in L$ such that $[t, x]\notin L_p$ (although it lies in L_{p-1} by* (a)).

(c) $L_1\neq 0$.

(d) *If S is an A-invariant subspace of L containing L_0 such that $[L_0, S]\subset S$, then either $S=L$ or $S=L_0$.*

Then we have

(1) $L_2=L_3= \cdots = L_k=0$.

(2) *$L=J+ \cdots +J$, where J is a simple Lie algebra.*

(3) *L can be decomposed as follows*:

$$L = F_{-1} + F_0 + F_1 \quad (\textit{vector space direct sum}),$$

in such a way that

$$L_1 = F_1, \qquad L_0 = F_0 + F_1,$$

$$[F_{-1}, F_{-1}] = 0, \qquad [F_{-1}, F_0] \subset F_{-1}, \qquad [F_{-1}, F_1] \subset F_0,$$

$$[F_0, F_0] \subset F_0, \qquad [F_0, F_1] \subset F_1, \qquad [F_1, F_1] = 0.$$

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