ON THE GROUP $\mathcal{E}[X]$ OF HOMOTOPY EQUIVALENCE MAPS

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Let X be a CW-complex; we shall consider the group[2]

$$\mathcal{E}[X]$$

formed by the homotopy classes of equivalence maps from X into itself with the operation induced by the composition of maps. It is clear to see that this group depends only on the homotopy type of X, hence should be determined by the known homotopy invariants of X. This is the problem which we shall try to study here. In fact, there exists a spectral sequence converging to $\mathcal{E}[X]$, whose initial terms are given, roughly speaking, by the cohomology of X and the automorphism group of its homotopy group.

Besides the satisfaction of curiosity, the group $\mathcal{E}[X]$ seems to have other interests. For example, it operates canonically on the special cohomology group [1] of X

$$\mathcal{E}[X] \times K(X) \to K(X)$$

and the quotient will be smaller than $K(X)$. In fact we can determine, more generally, the quotient

$$[X, G]/\mathcal{E}[X]$$

of the operation

$$\mathcal{E}[X] \times [X, G] \to [X, G]$$

where $[X, G]$ denotes the group of homotopy classes of maps from X into a topological group G. This may be considered as a first approach to determine the orbit space of the operation

$$(\mathcal{E}[X] \times \mathcal{E}[Y]) \times [X, Y] \to [X, Y]$$

where $[X, Y]$ is the set of homotopy classes of maps from X into Y.

The study of the group $\mathcal{E}[X]$ gives also some information about the image and the kernel of the canonical homomorphism (inversely, they determine the group $\mathcal{E}[X]$)

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[2] This group is also studied by M. Arkowitz, M. G. Barratt, C. R. Curjel, D. W. Kahn and P. Olum in the special case.