

RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

GROUPS WITH A BRUHAT DECOMPOSITION

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Introduction. In §1, we consider a group G which satisfies a simplified version of the axioms of Steinberg [3], and state some general results on the structure of G , the first two of which are due to J. Tits [7; 9]. The main result is stated in §2, and can be viewed as a generalization of a theorem of Higman and McLaughlin [2, Theorem 2]. The theorem states essentially that a finite simple group G , which satisfies certain structural assumptions independent of the arithmetical structure of G , and whose Weyl group is isomorphic to the Weyl group of a complex simple Lie algebra \mathfrak{g} of type A_n ($n \geq 2$), D_n ($n \geq 4$), or E_n ($n = 6, 7, 8$), is isomorphic to the group of Chevalley [1] determined by \mathfrak{g} and some finite field K . The possibility of such a theorem can be seen in the paper of Tits [8], who showed that a group with a root data is the amalgamated product of the canonically imbedded subgroups of rank two. For a group G satisfying the hypotheses of the theorem, the subgroups of rank two and their amalgamation are uniquely determined by the Weyl group. The applicability of the theorem is limited to the groups associated with simple Lie algebras of types A_n ($n \geq 2$), D_n , and E_n because these are the simple groups of Chevalley all of whose canonically imbedded simple subgroups of rank two are of type A_2 , and so can be classified by the result of Higman and McLaughlin.

1. The structure of groups with a Bruhat decomposition. Throughout this note, we shall use the following notations: $|A|$, cardinality of A ; $A \triangleleft B$, A is normal in B ; $\langle A, B, \dots \rangle$, group generated by A, B, \dots ; $C(A)$, center of A ; (A, B) , group generated by all commutators $(a, b) = aba^{-1}b^{-1}$, $a \in A, b \in B$.

The groups considered in §1 are not assumed to be finite.

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