

INFINITE-DIMENSIONAL IRREDUCIBLE REPRESENTATIONS OF COMPACT SEMI-SIMPLE GROUPS

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A great deal of progress has been made in the past two decades in the study, and especially the classification of, the unitary representations of the open semi-simple Lie groups. On the other hand, the classification problem is not so well posed or effectively definitive as might initially appear. The existing classifications consist roughly and for the most part of lists of inequivalent irreducible representations exhausting those in a given abstractly defined or otherwise cohesive category. Such lists can of course be extremely useful. However, the form in which the representations are explicitly given, the choice of concrete representation space, etc., is neither effectively unique nor immaterial.

For example, the work of Kunze and Stein giving a highly compact description of the representations in certain categories by means of analytic continuation in relevant parameters is based on a different presentation from that in the earlier literature on the representations in question. A different aspect which may be cited is that most classifications may be regarded as based on the existence of a maximal abelian algebra of operators left invariant by the representation. From this derives a representation in terms of the action of the group as a transformation group on the spectrum of a dense subalgebra, combined with a corresponding "multiplier." However, the question of the extent to which there exist other such algebras of imprimitivity, apart from the ones involved in the existing presentations, is largely unanswered.

On the whole, the general structure of the representations of the open semi-simple Lie groups has not yet been shown to possess the transparency and unique form which might be hoped for. The purpose of the present note is to describe an observation indicating possibilities for the classification of these representations directly in terms of local representations of the associated compact groups. It is of course too much to expect that the unitary representations of the compact groups should suffice; grossly speaking, there are simply too few of them. On the other hand, there is an apparently prevalent conception that there are no other interesting ones in Hilbert space, based on results indicating the similarity of various types of analyti-

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