A PROOF OF THE CORONA CONJECTURE FOR FINITE OPEN RIEMANN SURFACES[1]

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For an open Riemann surface X the corona conjecture is the following: let $B(X)$ be the algebra of bounded analytic functions on X and let $\mathfrak{M}(X)$ be the space of maximal ideals of $B(X)$; then X is dense in $\mathfrak{M}(X)$. Carleson [3] has proved that the corona conjecture is true for the open unit disk D. We will sketch a proof of the following extension of Carleson's Theorem.

THEOREM. *If X is a finite open Riemann surface, then X is dense in $\mathfrak{M}(X)$.*

By a finite open Riemann surface is meant a proper, open, connected subset of a compact Riemann surface W whose boundary Γ is also the boundary of $W-X$ and consists of a finite number of closed analytic arcs. Since $W-X$ has an interior we may employ the Riemann-Roch Theorem to show that $B(X)$ has enough functions to separate points and provide each point in X with a local uniformizer. Such a surface X therefore admits a natural homeomorphic imbedding into $\mathfrak{M}(X)$; thus the corona conjecture is seen to be meaningful.

Let X be a finite open Riemann surface. Ahlfors [1] has shown that there exists an analytic mapping p_0 of $\overline{X}$ into the plane such that $p=p_0|\overline{X}$ is an n-fold covering of X onto D and $p_0(\Gamma)=\overline{D}-D$. Since Γ consists of closed analytic arcs, no ramification occurs on $\overline{D}-D$. Clearly p^*, the adjoint of p, is a C-isomorphism of $B(D)$ into $B(X)$, C being the complex field. Let $B(D)^*$ denote the range of p^*, and for $f\in B(D)$ let $p^*(f)=f^*$.

Let σ_k denote the kth elementary symmetric function on n letters. For $z\in D$ let $p^{-1}(z)=\{x_1(z),\cdots,x_n(z)\}$, each appearing to its multiplicity. Given $f\in B(X)$, $\sigma_k(f(x_1(z)),\cdots,f(x_n(z)))$ is in $B(D)$. Thus, as is well known, $B(X)$ is integrally dependent on $B(D)^*$.

Given $N\in\mathfrak{M}(X)$ let $M^*=N\cap B(D)^*$ and let $P(N)=(p^*)^{-1}(M^*)$. Since $\mathfrak{M}(X)$ and $\mathfrak{M}(D)$ have the weak topology, P is continuous. Further, P is an extension of p. Since $B(X)$ is integrally dependent on $B(D)^*$, P is surjective. For $f\in B(D)((B(X))$ let $\hat{f}$ denote the natural extension of f to $\mathfrak{M}(D)((\mathfrak{M}(X))$. (See Hoffman [4, Chapter 10] for details.) Given $f\in B(D)$, $\hat{f}P=f^{*\frown}$. Let z denote the identity function

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