# RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

## ANALYTIC FUNCTIONS AND DIRICHLET PROBLEM[1]

BY G. LUMER

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In a recent paper, K. Hoffman [6], brings to a considerable degree of generality a certain part of the theory of analytic functions which in the classical context deals with the Hardy classes $H^p$. He also discusses analytic structures in the maximal ideal spaces of corresponding function algebras. For the setting of these general results, related to earlier work of Arens and Singer [1; 2], Helson and Lowdenslager [5], Bochner [4], and others, the concept of a logmodular algebra is introduced, generalizing that of Dirichlet algebra, and most of the results are valid in the latter situation. This development leans heavily, as Hoffman points out, on two fundamental properties of logmodular algebras:

(i) The representing measure (see [6]) associated with a point of the maximal ideal space is uniquely determined.

(ii) If $\mu$ is a representing measure, $A$ a logmodular algebra, then $A+\overline{A}$ is dense in $L^2(\mu)$.

The purpose of this abstract is to put in evidence the basic role played by (i) in the whole theory. Thus, for instance, it was not known whether conditions (i) and (ii) imposed on an arbitrary sup-norm algebra (see [6]) would be enough to recover in their full strength all the results valid for logmodular algebras. Actually, we shall show that (i)$\Rightarrow$(ii), and that the (additional) assumption of (i) alone yields all the results established in [6] for logmodular algebras. From a heuristic point of view this does not appear as an accident, since (i) is equivalent to the solvability of a certain Dirichlet problem associated with the algebra, to whose exact meaning we shall return below.

I. **Uniqueness of representing measures.** In what follows, $A$ shall always denote a sup-norm algebra, in the sense of [6], on the compact Hausdorff space $X$. $\mathfrak{M}$ shall denote the maximal ideal space of $A$, $\hat{A}$ the image of $A$ under the Gelfand representation; and for $m\in\mathfrak{M}$,

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