BOOK REVIEWS

Generalized functions and partial differential equations. By Avner Friedman. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963. xii+340 pp. $10.00.

This book deals with applications of Fourier transformation to certain problems of linear partial differential equations. Fourier transformation is meant here in the sense of the theory of distributions and of generalized functions. The author follows rather closely the original presentations of these theories by Laurent Schwartz and Gel'fand and Shilov. Chapter 1 provides the functional analysis background, giving the elements of the theory of topological vector spaces along the lines of N. Bourbaki, *Espaces vectoriels topologiques*, Paris, 1953–1955. Use is made of a subclass of $\mathfrak{F}$-spaces, under the name of complete countably normed. Perfect spaces are complete countably normed spaces in which every closed bounded set is compact. The reviewer has not found a single nontrivial property of the complete countably normed spaces which does not extend to $\mathfrak{F}$-spaces and it seems to him that this new definition complicates needlessly the picture. Chapter 2 introduces the two main classes of spaces of test functions. The spaces of the first kind, denoted by $K\{M_m\}$, consist of C^∞ functions ϕ on R^n satisfying inequalities of the kind

$$\sup_{|p|\leq m} \sup_{x} M_m(x) \mid D^p\phi(x) \mid < +\infty.$$

The weight functions $M_m(x)$ are allowed to be infinite on certain sets, in which $D^p\phi$ must then vanish; the sup with respect to x is performed over the complement of these sets. The elements of the spaces of the second kind, called $Z\{M_m\}$, are essentially entire functions on C^n, satisfying a sequence of inequalities

$$\sup_{z\in C^n} M_m(z) \mid \phi(z) \mid < +\infty.$$

Here the M_m form a nondecreasing sequence of positive continuous functions in C^n. Further spaces of test functions are obtained by taking the intersections and the inductive limits of the spaces above.

Chapter 2 proceeds then with the definition of generalized functions and the study of their basic properties. A generalized function is a continuous linear form over a space of test functions. A first example is provided by the distributions; the main spaces of test functions for distributions, the spaces $\mathfrak{D}$, $\mathcal{E}$, $\mathcal{S}$ are of the type $K\{M_m\}$ (or inductive limits of spaces $K\{M_m\}$). Chapter 3 constitutes a sum-