FOURIER SERIES IN SEVERAL VARIABLES

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0. Preface. This article is a survey of certain aspects of the theory of multiple Fourier and trigonometric series. It is by no means meant to be a complete survey; for example, it is practically disjoint with the material covered on the subject in Zygmund's book [**38**, Chapter 17].

There are eight sections to this survey. §1 is the introduction. §2, §3, and §4 are expository in the sense that the main theorems in each section are proved. §5, §6, and §7 are descriptive. §8 consists of two bibliographies, a bibliography for the survey itself, and a general bibliography.

§2 deals with the now classical theory of the Bochner-Riesz summability of multiple Fourier series and the Abel summability of multiple Fourier series. §3 presents Bochner's counter-example for the critical index in summability theory in considerable detail. §4 is concerned with the uniqueness of multiple trigonometric series and proves the main theorem in the subject so far, i.e. uniqueness under Abel summability (due to the present author). §5 describes some results in conjugate multiple Fourier series defined by means of the Calderón-Zygmund kernel and related topics, i.e. analyticity in several variables. §6 deals with the Riemannian theory of multiple trigonometric series. §7 describes some applications to geometric integration theory and potential theory.

1. Introduction. Operating in k-dimensional Euclidean space, E_k, $k \geqq 2$, we shall use the following notation:

$$x = (x_1, \cdots, x_k), \qquad y = (y_1, \cdots, y_k),$$
$$\alpha x + \beta y = (\alpha x_1 + \beta y_1, \cdots, \alpha x_k + \beta y_k),$$
$$(x, y) = x_1 y_1 + \cdots + x_k y_k, \qquad |x| = (x, x)^{1/2},$$
$$m = (m_1, \cdots, m_k), \qquad (m, x) = m_1 x_1 + \cdots + m_k x_k.$$

With $f(x)$ a function in L^1 on T_k, the k-dimensional torus

$$\{x; -\pi < x_j \leqq \pi, j = 1, \cdots, k\}$$

and m an integral lattice point, we shall designate the series

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