NONLINEAR PARABOLIC BOUNDARY VALUE PROBLEMS OF ARBITRARY ORDER[1]

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In a number of recent papers the writer has developed a new nonlinear version of the orthogonal projection argument to prove the existence of solutions of variational type for boundary value problems for nonlinear partial differential equations, first for nonlinear elliptic equations in [2], [3], [4], [5], and [6] and more recently for nonlinear parabolic equations in [7]. This new method is based on general theorems on the existence of solutions of equations in Hilbert space or reflexive Banach spaces involving operators satisfying very weak continuity conditions but having suitable monotonicity properties.[2] Our results in [6] in the elliptic case included much stronger versions of theorems of Visik ([12], [13], [14]) on solutions of boundary value problems for equations of the form

$$Au = \sum_{|\alpha| \leq m} D^\alpha A_\alpha(x, u, \cdots, D^m u) = f(x), \tag{1}$$

with the A_α having polynomial growth in $(u, \cdots, D^m u)$.[3] In the parabolic case, we considered in [7] mixed initial-boundary value problems for equations of the form

$$\frac{\partial u}{\partial t} + \sum_{|\alpha| \leq m} D^\alpha A_\alpha(x, t, u, \cdots, D^m u) = f(x, t) \tag{2}$$

with A_α having at most linear growth in $(u, \cdots, D^m u)$.

In the present note, we present in summary a complete generalization of the treatment of the parabolic problems to the case of A_α of polynomial growth in $(u, \cdots, D^m u)$. The detailed arguments will appear in [8]. This generalization is based upon a stronger theorem in reflexive Banach spaces, Theorem 2 below, which generalizes the abstract theorems given in both [6] and [7].

We use the notation and definitions of [6] and [7]. We let Ω be an

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[2] Earlier but weaker results on continuous monotone operator equations were obtained by Vainberg and Kachurovski [11] and G. J. Minty [10].

[3] For the case of linear growth and the Euler equations of a regular variational problem, an interesting generalization of the Morse Theory has been obtained recently by S. Smale and by Smale and R. Palais.