ON THE GREEN'S FUNCTION FOR SECOND ORDER PARABOLIC DIFFERENTIAL EQUATIONS WITH DISCONTINUOUS COEFFICIENTS[1]

BY D. G. ARONSON

Communicated by Louis Nirenberg, April 15, 1963

1. **Introduction.** We consider the second order linear parabolic differential operator in divergence form

$$Lu \equiv \frac{\partial u}{\partial t} - \frac{\partial}{\partial x_j}\left\{a_{ij}(x, t)\frac{\partial u}{\partial x_i}\right\}$$

for $(x, t) \in Q_T \equiv \Omega \times (0, T]$, where Ω is a bounded open simply connected region in E^n $(n \geqq 2)$, T an arbitrary positive number, and the a_{ij} are assumed only to be bounded and measurable in $\overline{Q}_T$. The purpose of this note is to report results concerning the existence and certain properties of the generalized Green's function $g(x_0, t_0; x, t)$ for L subject to homogeneous boundary conditions. Our results are largely based on the maximum principle for the problem

$$\begin{aligned} Lu &= \operatorname{div} \boldsymbol{f}(x, t) - f(x, t) \text{ in } Q_T; \\ u(x, t) &= \psi(x, t) \text{ on } \Gamma \equiv \overline{S} \cup (\Omega \times \{t = 0\}), \end{aligned} \tag{1.1}$$

where $S \equiv \partial\Omega \times (0, T]$ and f, $\boldsymbol{f}$, ψ are given functions. This maximum principle, which we discuss in §2, is a generalization of the maximum principle proved by Ladyženskaja and Ural'ceva [3] since we make less restrictive assumptions on the inhomogeneous terms in (1.1). In §3 we introduce the corresponding generalizations of the Hölder continuity, existence and uniqueness theorems of [3]. Our main results on the existence and properties of the Green's function are given in §§4, 5 and 6. The proofs of the theorems which we state below, as well as extensions of our results to equations with lower order terms and to the case of nonzero boundary conditions will be given elsewhere. The author is indebted to Professor Guido Stampacchia who first introduced him to many of the ideas embodied in this work, and to Professor Hans Weinberger for many stimulating discussions in the course of its preparation.

2. **The maximum principle.** We shall always assume that the coefficients a_{ij} of L are measurable in Q_T and that there exists a constant

[1] This work was supported in part by the Office of Naval Research under Contract Nonr-710(16), (NR 043 041).