FUNDAMENTAL SOLUTIONS OF INVARIANT DIFFERENTIAL OPERATORS ON SYMMETRIC SPACES

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1. Introduction and notation. Let S be a Riemannian globally symmetric space, G the largest connected group of isometries of S in the compact open topology. We assume that S is of the noncompact type, that is, G is semisimple and has no compact normal subgroup $\neq \{e\}$. Let o be any point in S, K the isotropy subgroup of G at o, $\mathfrak{k}$ and $\mathfrak{g}$ their respective Lie algebras, and $\mathfrak{p}$ the orthogonal complement of $\mathfrak{k}$ in $\mathfrak{g}$ with respect to the Killing form B of $\mathfrak{g}$. Let $\mathfrak{a}$ be any maximal abelian subspace of $\mathfrak{p}$ and let $A=\exp(\mathfrak{a})$. For each λ in the dual space of $\mathfrak{a}$ (which we identify with $\mathfrak{a}$, via B) let $\mathfrak{g}_\lambda=\{X\in\mathfrak{g} \mid [H,X]=\lambda(H)X \text{ for all } H\in\mathfrak{a}\}$. Let $d_\lambda=\dim(\mathfrak{g}_\lambda)$. Choose some order on $\mathfrak{a}$ and let

$$\mathfrak{n}=\sum_{\lambda>0}\mathfrak{g}_\lambda, \qquad \rho=\sum_{\lambda>0} d_\lambda\lambda, \qquad \pi'=\prod_{\lambda>0}\lambda^{d_\lambda}$$

and let π denote the product of the distinct prime factors in π'. Then we have the Iwasawa decompositions $\mathfrak{g}=\mathfrak{k}+\mathfrak{a}+\mathfrak{n}$, $G=KAN$ where N is the nilpotent group $\exp(\mathfrak{n})$. Given $g\in G$, let $H(g)$ denote the unique element in $\mathfrak{a}$ for which $g\in K \exp H(g)N$. Let W denote the Weyl group M'/M where M and M', respectively, denote the centralizer and normalizer of $\mathfrak{a}$ in K.

For each $\lambda\in\mathfrak{a}$ consider the spherical function

$$\phi_\lambda(x)=\int_K e^{(i\lambda-\rho)(H(xk))}dk \qquad (x\in G)$$

dk being the normalized Haar measure on K. Let $c(\lambda)$ denote Harish-Chandra's function on $\mathfrak{a}$ which occurs in the leading term of the asymptotic expansion of ϕ_λ [2, p. 283], i.e.,

$$\phi_\lambda(\exp H)\sim\sum_{s\in W}c(s\lambda)e^{(is\lambda-\rho)(H)}$$

where λ and H are suitably restricted in $\mathfrak{a}$.

Each $x\in G$ can be written uniquely in the form $x=k\exp X (k\in K, X\in\mathfrak{p})$. We put $|X|=(B(X,X))^{1/2}$ and $\omega(X)=\{\det(\sinh ad X/ad X)_\mathfrak{p}\}^{1/2}$ where the subscript $\mathfrak{p}$ indicates restriction to $\mathfrak{p}$ of the linear

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