BOOK REVIEW

Topologische lineare Räume, Bd. 1. By Gottfried Köthe. (Die Grundlehren der Mathematischen Wissenschaften, Bd. 107.) Springer, Berlin, 1960. xii+456 pp.

It seems to be a commonplace nowadays that the concept of a vector space of finite or infinite dimension is an indispensable part of the education of every young mathematician whether pure or applied as well as of the theoretical physicist (or even the theoretical engineer, as some people already claim). Yet not too long ago, within the past forty years or so, this concept was unfamiliar or unknown (or at the very least not as fundamental as it appears today) to various very good mathematicians for whom a vector space was simply R^n or C^n. Hilbert spaces and later Banach spaces have progressively become ever more useful mathematical concepts, and geometrical intuition as applied to analysis has climbed from $n=1, 2, 3$, etc. to $n=\infty$. In this way, functional analysis became a well-established branch of mathematics. Even more recently, topological vector spaces as well have turned out to be basic to the degree that they have become a must in the education of any graduate student of mathematics.

At the present time, problems in analysis are treated on the dual basis of "hard" analysis, depending on the fine concepts and techniques of integration theory, analytic functions, differential equations, potential theory, harmonic analysis, etc. on R^n and C^n, and of the methods of "soft" analysis, namely those of Hilbert spaces, Banach spaces, topological vector spaces, their analogues for algebras and modules, etc. Of course, such a sharp boundary line between hard and soft analysis is tending to disappear. In some advanced mathematical communities it has indeed been forgotten, but some groups still stick to a firm distinction between these two tendencies as a matter of personal pride.

In the history of the ideas, methods, and applications of topological vector spaces up to the present time, there are two individuals whose names are landmarks in the development of this theory, namely Stephan Banach in connection with normed vector spaces and Laurent Schwartz in topological vector spaces. This is true not only because their work forms a very substantial contribution to the real progress of this branch of mathematics but also because they directly inspired many significant contributions in this area by their colleagues and students. It is interesting to read now what André Weil