A TOPOLOGICAL CLASSIFICATION OF CERTAIN 3-MANIFOLDS

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Introduction. In [1] J. Stallings proves that members of the class of closed irreducible 3-manifolds which are fibered over a circle by an aspherical 2-manifold may be distinguished from other closed irreducible 3-manifolds by their fundamental group alone.

He asks whether two members of this class of 3-manifolds are homeomorphic if they have isomorphic fundamental groups. This question is answered in the affirmative here, thus giving a classification of these manifolds according to their fundamental group.

The closed case. Let us denote by $\mathfrak{M}$ the class of all 3-manifolds satisfying the following conditions:

(a) Manifolds of $\mathfrak{M}$ are irreducible (every 2-sphere bounds a 3-cell).

(b) Manifolds of $\mathfrak{M}$ are closed.

(c) Manifolds of $\mathfrak{M}$ have fundamental groups which contain a finitely generated normal subgroup of order >2, with quotient group an infinite cyclic group.

THEOREM 1. *Let M_2 be any closed irreducible 3-manifold. Let M_1 belong to $\mathfrak{M}$, then M_1 is homeomorphic to M_2 if and only if $\pi_1(M_1)$ is isomorphic to $\pi_1(M_2)$.*

PROOF. One direction is trivial. By Stallings theorem [1] M_1 admits a fibering over S^1, with fiber a closed 2-manifold T_1. Let

$$1 \to H_1 \to G_1 \to Z \to 0 \tag{1}$$

denote the sequence of fundamental groups of T_1, M_1, S^1, respectively corresponding to this fibering. Let ρ^* denote the assumed isomorphism from $\pi_1(M_1) = G_1$ to $\pi_1(M_2) = G_2$. Then ρ^* induces

$$1 \to H_2 \to G_2 \to Z \to 0. \tag{2}$$

Now, G_1 and G_2 are both described by giving the automorphisms ϕ_1^*, ϕ_2^* of H_1, H_2, which are induced by a generator of Z, pulled back to G_1, G_2, and then acting on H_1, H_2 by conjugation.

Since ρ^* is an isomorphism we may assume

$$\rho^*\phi_1^* = \phi_2^*(\rho^* \mid H_1). \tag{3}$$