POLYHEDRAL NEIGHBORHOODS IN TRIANGULATED MANIFOLDS

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This note is an outline of some of the author's recent work concerning triangulated manifolds. A combinatorial structure is never assumed; indeed with this condition added, our results are mostly corollaries to well known theorems. The purpose of these investigations is two-fold: it is possible that they may lead to a proof of the cellularity of vertex stars in manifolds (a result that would have critical implications for the theory); but also, should a noncombinatorial triangulation of a manifold be found, they might serve as a starting point for the local study of such examples.

Our tools are:

I. The generalized Schoenflies theorem of Brown and Mazur [**2**; **3**].

II. Let M be a compact Hausdorff space which is the union of two open sets each of which is a homeomorph of E^n; then M is homeomorphic to S^n (we write $M \approx S^n$). This is an immediate consequence of I.

III. If the cone over Y $(=C(Y))$ is n-euclidean at the vertex, then the suspension of Y $(=S(Y))$ is topologically S^n. This proposition of Mazur [**3**] follows from II.

The join of spaces X and Y is written $X \circ Y$. The kth barycentric subdivision of a polyhedron P is denoted by ${}^kP$. Let (K, L) be a polyhedral pair. The *stellar neighborhood of L in K* $(=N(K, L))$ is the union of all open simplexes of K with vertices in L. The closure of $N(K, L)$ is represented by $\mathrm{St}(K, L)$ (read star in K of L). For a simplex w in K let $\mathrm{Lk}(K, w)$ be the link of w in K, and $\mathrm{Cl}(K, w)$ $(=w \circ \mathrm{Lk}(K, w))$ be the cluster of w in K. For a simplex $w = u \circ v$ let D be the set of midpoints of segments from u to v and let $B(w, u)$ be the union of all straight segments $x \circ p$ in w with $x \in u$ and $p \in D$. If L is full in K define the *barrel neighborhood $B(K, L)$ of L in K* as the union of all sets $B(w, u)$ with w and u simplexes of $\mathrm{St}(K, L)$ and L, respectively.

If K is homogeneous (in the sense of [**1**]) then the double of K, or $2K$, consists of K and a disjoint copy K' with their combinatorial boundaries canonically identified. A quotient space of X whose only possible nondegenerate element is Y will be written X/Y. A subset A of an n-manifold is *cellular* if it is the intersection of n-cells (C_i)

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