PONTRJAGIN NUMBERS OF MAPS

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We consider an integral valued K-genus; that is, a ring homomorphism $K: \Omega \to Z$ which carries the unit of the oriented Thom bordism ring into 1. The integrality of certain genera, such as the A-genus or the L-genus [6, p. 13], provides some divisibility conditions which must be satisfied by the Pontrjagin numbers of closed manifolds. For the study of the oriented bordism module $\Omega_*(X)$ of a space the Pontrjagin numbers of a map of a closed oriented manifold into X were defined in [4]. We shall explore in this note the corresponding divisibility conditions, caused by the integrality of K, which must be satisfied by the Pontrjagin numbers of such maps.

For each space X let $Q(X)$ be the collection of all additive rational valued homomorphisms $\Gamma: \Omega_*(X) \to Q$ which are compatible with the K-genus in the sense that for $[M^n, f] \in \Omega_n(X)$, $[V^n] \in \Omega_m$, $\Gamma([M^n, f][V^m]) = (\Gamma([M^n, f]))(K([V^m]))$. The $Q(X)$ is a linear space over the rationals. Let $K_j(p_1, \cdots, p_j)$ be the multiplicative sequence of homogeneous polynomials with rational coefficients which determine K [6, p. 80]. For each closed oriented manifold M^n, let $k(M^n) \in H^*(M^n; Q)$ be the cohomology class $(1, K_1(p_1), K_2(p_1, p_2), \cdots, K_j(p_1, \cdots, p_j), \cdots)$, where $p_j \in H^{4j}(M^n; Q)$ is a Pontrjagin class of the tangent bundle to M^n. We define a linear homomorphism $\Psi: H^{**}(X; Q) \to Q(X)$. For $c \in H^{**}(X; Q)$ we set $\Gamma_c([M^n, f]) = \langle k(M^n)f^*(c), \sigma_n \rangle$, where σ_n is the orientation class and the brackets denote the cap product $\sigma_n \cap k(M^n)f^*(c)$ followed by the augmentation $\epsilon_*: H_*(X; Q) \to Q$. A CW-complex X is of finite type if and only if every skeleton $X^{(k)}$ is finite. We shall only consider such spaces.

1. *If X is a* CW-*complex of finite type then $\Psi(c) = \Gamma_c$ is a linear isomorphism between $H^{**}(X; Q)$ and $Q(X)$.*

A $\Gamma \in Q(X)$ is integral valued if and only if $\Gamma(\Omega_*(X)) \subset Z$. This additive subgroup $z(X) \subset H^{**}(X; Q)$ consists of cohomology classes c for which $\Psi(c)$ is integral valued. For each space the subgroup $z(X)$, which is not always homogeneous, represents the divisibility conditions imposed by K on the Pontrjagin numbers of maps of closed oriented manifolds into X. Another interpretation is the consideration of $z(X)$ as the group of virtual K-genera on X [6, p. 85]. Under the homomorphism induced by $f: X \to Y$, $f^*(z(Y)) \subset z(X)$. If

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