A BORDISM THEORY FOR ACTIONS OF AN ABELIAN GROUP

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1. **Introduction.** This note is a preliminary sketch of a general bordism theory for the differentiable actions of a finite abelian group on closed manifolds. The present note is based upon the techniques outlined in [1] for the study of differentiable periodic maps. We fix a finite abelian group A and in A we choose a family K of subgroups. We assume that any subgroup of an element in K is also an element in K. We wish to consider all differentiable actions (A, B^n) on compact manifolds (possibly with boundary) which have the property that each isotropy group A_x is an element of K. Two such actions are strictly equivalent if and only if they are connected by an equivariant diffeomorphism.

We now describe the equivariant bordism theory. An action (A, M^n) on a closed manifold, all of whose isotropy groups lie in K, is said to equivariantly bord if and only if there is an (A, B^{n+1}), all of whose isotropy groups also belong to K, for which the induced action on the boundary $(A, \partial B^{n+1})$ is equivariantly diffeomorphic to (A, M^n). From two actions (A, M_1^n) and (A, M_2^n) a disjoint union action may be formed $(A, M_1^n \cup M_2^n)$ with $M_1^n \cap M_2^n = \emptyset$, and with A restricted to M_i^n equal to (A, M_i^n) for $i=1, 2$. We shall say that (A, M_1^n) is equivariantly bordant to (A, M_2^n) if and only if their disjoint union equivariantly bords. Again we recall that every isotropy group is to be a member of the family K. We have defined an equivalence by introducing the equivariant bordism relation. The proof of transitivity is based on an equivariant collaring theorem which asserts that for any differentiable (A, B^{n+1}) there is an open invariant $U \supset \partial B^{n+1}$ and an equivariant diffeomorphism $m: (A, U) \to (A, \partial B^{n+1} \times [0, 1))$ for which $m(x) = (x, 0)$, $x \in \partial B^{n+1}$, and where $(A, \partial B^{n+1} \times [0, 1))$ is given by $\alpha(x, t) = (\alpha(x), t)$. We denote the unoriented bordism class of (A, M^n) by $[A, M^n]_2$ and the collection of all such equivalence classes by $I_n(A; K)$. An abelian group structure, in which every element has order 2, can be imposed on $I_n(A; K)$. We shall exhibit the basic fact that this is a finite group. On the weak direct sum $I_*(A; K) = \sum_0^\infty I_n(A; K)$ we can impose a graded right module structure over the unoriented Thom bordism ring $\mathfrak{N}$. For

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