fies (F) with $\{(A_1, B_1), \cdots, (A_k, B_k)\}$ as a class of order-pairs. The $2k+1$ integers $k^2, \cdots, k^2+2k$ are reversed by ρ, but two of them must fall in the same set A_i. This is a contradiction.

Therefore G is a proper subgroup of S_∞.

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ON THE ISOMORPHISM PROBLEM FOR BERNOULLI SCHEMES

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Communicated by J. L. Doob, November 27, 1962

1. DEFINITION 1. A Bernoulli scheme $(E, \Omega, \mathfrak{F}, P, T)$ is a probability space together with a transformation T, where

(i) $E=\{1, \cdots, n\}$ for some positive integer n, or $E=\{1, 2, \cdots\}$,

(ii) $\Omega=\{\omega=(\cdots, \omega_{-1}, \omega_0, \omega_1, \cdots) \mid \omega_i \in E \text{ for all } i\}$,

(iii) $\mathfrak{F}$ is the smallest σ-algebra containing all sets $A_i^k=\{\omega \mid \omega_i=k\}$,

(iv) $q_k>0$ is defined for $k\in E$ with $\sum_{k\in E} q_k=1$, P is the product measure on $\mathfrak{F}$ defined by $P\{A_i^k\}=q_k$ for all i,

(v) T is the shift transformation defined on Ω, i.e., $T\omega=\omega'$ if and only if $\omega_i'=\omega_{i+1}$ for all i.

We shall sometimes refer to a Bernoulli scheme as a $(q_1, \cdots, q_n)$-scheme or a $(q_1, q_2, \cdots)$-scheme depending upon whether $E=\{1, \cdots, n\}$ or $E=\{1, 2, \cdots\}$.

DEFINITION 2. Two Bernoulli schemes $(E, \Omega, \mathfrak{F}, P, T)$ and $(E', \Omega', \mathfrak{F}', P', T')$ are said to be *isomorphic modulo sets of measure zero* (or simply *isomorphic*) if there exist sets $D\in\mathfrak{F}$, $D'\in\mathfrak{F}'$ and a mapping $\phi: D\rightarrow D'$ such that

(i) $TD=D$,

(ii) $\phi: D\rightarrow D'$ is one-to-one and onto,

(iii) $\phi(T\omega)=T'(\phi\omega)$ for all $\omega\in D$,

(iv) if $A\subset D$ then $A\in\mathfrak{F}$ if and only if $\phi A\in\mathfrak{F}'$,

(v) if $A\subset D$ and $A\in\mathfrak{F}$ then $P(A)=P'(\phi A)$,

(vi) $P(D)=1$.

DEFINITION 3. The *entropy* of a $(q_1, \cdots, q_n)$-scheme $[(q_1, q_2, \cdots)$-scheme$]$ is given by

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[1] Research supported by the National Science Foundation, Grant NSF G-21219.

[2] Research done under the auspices of the United States Atomic Energy Commission.