RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

CONVOLUTION OF SEQUENCES[1]

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Communicated by Edwin Hewitt, October 29, 1962

A summability method is a linear functional on a space of sequences $\mathcal{S}$. The method ϕ is said to be regular if, for each convergent sequence $s=\{s_n\}$ we have $\phi(s)=\lim_{n\to\infty} s_n$. In this paper we define various types of convolution (multiplication) of sequences; we use the symbol $*$ to denote convolution. Our convolution is always distributive, but not necessarily associative or commutative. We consider the regular methods ϕ such that $\phi(s*t)=\phi(s)\phi(t)$ for all sequences s and t in the domain of ϕ, $\mathcal{S}(\phi)$, that is, the regular homomorphisms from $\mathcal{S}(\phi)$ to the real numbers. We write $\hat{s}(\phi)=\phi(s)$ for each sequence s in $\mathcal{S}(\phi)$ and we impose the weak topology on the set of homomorphic methods. In case the multiplication is commutative and associative and we were dealing with complex sequences, then $\mathcal{S}(\phi)$ would be a complex Banach algebra, $\hat{s}(\phi)$ would be the Fourier transform of the sequence s, and the weak topology on the set of homomorphic methods would yield the maximal ideal space of $\mathcal{S}(\phi)$. Although we shall deal with real sequences, we shall use a certain amount of Gel'fand theory.

The types of convolution to be considered are:

(a) Pointwise multiplication—if s and t are two bounded sequences then $s*t=\{s_nt_n\}$.

(b) Cauchy multiplication—if s and t are two sequences such that

$$S(z)=\sum_{n=0}^{\infty}a_nz^n,\quad T(z)=\sum_{n=0}^{\infty}b_nz^n,\quad (a_n=s_{n+1}-s_n,\ b_n=t_{n+1}-t_n)$$

are analytic and bounded in the unit circle D in the complex z-plane, then $s*t=\{\sum_{k=0}^{n}\sum_{j=0}^{k}a_jb_{k-j}\}$. We note that the power corresponding to $s*t$ is $S(z)T(z)$.

(c) If s and t are bounded sequences, and $B=(b_{nk})$ is a positive

[1] This research was supported by Nonr (710)16.