EXAMPLES OF PERIODIC MAPS ON EUCLIDEAN SPACES WITHOUT FIXED POINTS

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Let T be a map of period r on a Euclidean space E^n. Smith seems to have been the first to consider fixed points of T. He showed that T has a fixed point if r is a prime in [**4**], extended this result to r a power of a prime, and raised the question concerning the existence of a fixed point for r not a prime power in [**5**]; also cf. Problem 33 in [**3**]. Conner and Floyd gave an example of a contractible manifold M_r for every r not a prime power, and a map T of period r on M_r without fixed points [**2**]. They conjectured that M_r was a Euclidean space. This note shows that a slight modification of their example is Euclidean, hence:

THEOREM. *If r is an integer which is not a power of a prime, then there exists a triangulation τ of E^{9r}, a map T of period r on E^{9r} without fixed points, and T is simplicial relative to τ.*

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Preliminaries. Let K be a subcomplex of a Euclidean space E under a triangulation σ. Let $\sigma_K^{(1)}$ be the subdivision of σ obtained by adding barycenters of all simplexes not contained in K, cf. [**6**, p. 251]. $\sigma_K^{(i+1)} \equiv (\sigma_{iK}^{(i)})_K^{(i)}$. If K is the empty complex, $\sigma_K^{(i)} \equiv \sigma^{(i)}$, the usual ith barycentric subdivision. Denote the closed star of K in σ by $V(K, \sigma)$ and let $V^2(K, \sigma) = V(V(K, \sigma), \sigma)$. $N_W(K, \sigma) \equiv V(K, \sigma_K^{(2)})$ is a "regular" neighborhood of K; cf. [**6**, p. 293]. If K is a contractible finite subcomplex having dimension m and $E = E^n$, where $n \geqq 2m+5$, then it follows from Corollary 3 in [**6**, p. 298] that $N_W(K, \sigma)$ is an n-cell. Much use is made of this fact; however it will be convenient later to use the following neighborhood: $N_1(K, \sigma) \equiv V(K^{(2)}, \sigma^{(2)})$, i.e. the star of K (subdivided twice barycentrically) in $\sigma^{(2)}$. Since it will be necessary to use Whitehead's result, but only in a topological way (i.e. noncombinatorial), it suffices to show that $N_W(K, \sigma)$ and $N_1(K, \sigma)$ are homeomorphic. This can be done by looking at an n-simplex ρ in the triangulation $\sigma_K^{(1)}$ which intersects K, and constructing a canonical homeomorphism of $N_W(K, \sigma) \cap \rho$ and $N_1(K, \sigma) \cap \rho$ in such a way that two such homeomorphisms match on p-faces, $p < n$. Let $\rho = \rho_0 \circ \rho_1$,

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