ON CONTINUITY OF INVERSE OPERATORS

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This paper concerns the so-called "closed graph theorem" or "open mapping theorem" (cf. [**2**; **3**; **5**; **6**]). We intend to introduce here only the main result. Discussion is restricted to some general indications as to applications of the theorem proved. The full paper will be published in Studia Mathematica.

Suppose we have been given countable sets T, S, a subset N of the set T^S of all T-valued functions defined on S and a family of linear spaces with pseudonorms[2] $X_{p,q}$, $|\cdot|_{p,q}$, where (p, q) runs over the union of all graphs from N and all $X_{p,q}$ are linear subspaces of the same linear space X. Denote briefly $(N, X_{p,q}, |\cdot|_{p,q})$ by $\mathfrak{F}$. Having fixed $\mathfrak{F}$ we associate with each $(p_q)\in N$ a linear space with pseudonorm defined as follows:

$$X_{(p_q)} = \bigcap_{q\in S} X_{p_q,q}; \qquad |x|_{(p_q)} = \sum_{n=1}^{\infty} 2^{-n}|x|_{p_{q_n},q_n}(1 + |x|_{p_{q_n},q_n})^{-1},$$

where $S=q_1, q_2, \cdots$. Assuming $X=\bigcup_{(p_q)\in N} X_{(p_q)}$ we introduce $\mathfrak{F}$-boundedness in X as follows: a sequence $(x_n)\subset X$ is said to be *$\mathfrak{F}$-bounded* if (x_n) is bounded in at least one $X_{(p_q)}$, $|\cdot|_{(p_q)}$ in the usual sense. Suppose there is given a BC topology (τ) in X.[3] The topology (τ) *is said to be represented by* $\mathfrak{F}$ if the notions of $\mathfrak{F}$-boundedness and (τ)-boundedness coincide.

If for given $\mathfrak{F}$ there exists at least one BC topology that is represented by $\mathfrak{F}$, then this topology must be given by considering a pseudonorm being continuous on X iff it is continuous restricted to any $X_{(p_q)}$, $|\cdot|_{(p_q)}$, with $(p_q)\in N$. Therefore there is at most one BC topology that is represented by given F and we denote this topology by $(\tau_{\mathfrak{F}})$.

A locally convex topology (τ) is said to be *(b)-complete* if each bounded mapping of an h-normed[4] space Z into X can be extended to the completion of Z. (b)-completeness follows from sequential completeness, and remains after bornologic fortification of the initial topology.

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[2] Pseudonorms are not necessarily homogeneous.

[3] BC = bornologic locally convex.

[4] h-norm = homogenous norm.