ON A PROBLEM OF P. A. SMITH[1]

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1. **Introduction.** Throughout this note, Z_2 denotes the group of integers mod 2 and cohomology means the Alexander-Wallace-Spanier cohomology with coefficients in Z_2. By a *cohomology projective n-space* we mean a compact Hausdorff space Y whose cohomology ring $H^*(Y)$ is isomorphic to that of the real projective n-space. In [2], Smith proved that if Z_2 acts effectively on the real projective n-space such that the fixed point set $F(Z_2)$ is nonempty, then $F(Z_2)$ has exactly two components A_1 and A_2, where A_i is a cohomology projective n_i-space $(i=1, 2)$ and $n_1+n_2=n-1$. Smith then asked whether the result is true if the real projective n-space is replaced by a cohomology projective n-space. The purpose of this note is to give a positive answer to the question.

We wish to point out that the inclusion of ring structure in the definition of a cohomology projective n-space is indispensable as we may see from the following example. Let Y be the one-point union of a 1-sphere S^1 and a 2-sphere S^2. Clearly $H^*(Y)$ as a group is the same as the cohomology group of a projective plane. Let T be a generator of Z_2 and define the action of T on Y such that on S^i it is the reflexion with respect to the diameter passing through the point of contact. Then the fixed point set consists of three isolated points.

2. **A construction.** The proof of Smith's theorem in [2] has used the fact that a projective n-space admits an n-sphere as its two-folded covering space. It is therefore quite natural to expect that a cohomology projective n-space Y admits a cohomology n-sphere as its two-folded covering space. In the following we give a construction of such a cohomology n-sphere which is very similar to the construction of a covering space of a pathwise connected, locally pathwise connected, and locally pathwise simply connected space, with the dual of $H^1(Y)$ playing the role of fundamental group.

Let Y be a connected compact Hausdorff space and let $\alpha \in H^1(Y)$ be a nonzero element. Let $f\colon Y^2 \to Z_2$ be a 1-cocycle representing α; then there exists an open covering $\mathcal{V}$ of Y such that

$$f(y_0, y_2) = f(y_0, y_1) + f(y_1, y_2) \quad \text{whenever} \quad y_0, y_1, y_2 \in V \in \mathcal{V}.$$

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