ABSTRACT CLASS FORMATIONS

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Let k_0 be any field, Ω a normal separable extension of it, and $R=R(k_0, \Omega)$ the set of all fields of finite degree over k_0 which are contained in Ω. A class formation is a family of groups $\{E(K)\}_{K\in R}$, indexed by R, together with a family of monomorphisms $\Phi_{K,L}: E(K)\rightarrow E(L)$, defined for $K\subset L$, satisfying the Artin-Tate-Kawada axioms [1]. These axioms state that every automorphism of a $K\in R$ which is identity on k_0 defines an automorphism of $E(K)$; that these automorphisms commute in a reasonable way with the $\Phi_{K,L}$; that if k, $K\in R$ and k is fixed point field for a finite group H of automorphisms of K then

$$(1) \qquad (E(K))^H = \Phi_{k,K}E(k),$$

$(E(K))^H$ denoting the subgroup of $E(K)$ on which H acts simply; and that

$$(2) \qquad H^1(G, E(K)), = 0; \qquad H^2(G, E(K)) \text{cyclic of order } \#G$$

for all $K\in R$, all finite groups G of automorphisms of K/k_0, where $\#G$ denotes the number of elements of G. From well-known results of J. Tate [2] it follows that (2) is equivalent to

$$(2') \qquad H^r(G, E(K)) \cong H^{r-2}(G, Z) \qquad \text{for all } r,$$

where Z denotes the additive group of ordinary integers with G acting simply on it.

The best known examples of class formations are furnished by local and global class field theory but Kawada has found several others. In all these examples the groups $E(K)$ are defined by use of the structure of the fields K, so that by use of the class formation one can deduce connections between the arithmetic of the fields $K\in R$ and the algebraic extensions of these fields.

In this note we entirely ignore the structure of the fields $K\in R$, asking merely whether, for given R, there exists *any* family of groups indexed by R which form a class formation. Thus we use the fields in R merely as indices. Indeed, since the fields in R are in one-one correspondence with the open subgroups of the topological Galois group G_∞ of Ω/k_0 we could formulate our question in the following more general way: Given a topological group G_∞ which is projective limit of a directed system of finite groups does there exist a family of groups, indexed by the set of all open subgroups of G_∞, which