A GENERAL APPROACH TO BOUNDARY PROBLEMS[1]

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1. **Introduction.** In this paper we apply the method of negative norms to arbitrary boundary problems. The method has proved very powerful in dealing with elliptic problems (cf. [9]). Here we show that it is just as successful in general, without reference to the type of equation or boundary condition.

In the next section we define several negative norms and state two important representation theorems. Everything is done within the L^p framework, $1<p<\infty$. In §3 we give necessary and sufficient conditions for certain boundary problems to have solutions. In §4 it is noted that these theorems give new results even in the case of elliptic problems. In §5 it is shown how our methods can be employed to give complete aswers for the Višik-Sobolev problems.

Proofs of all our theorems will appear elsewhere.

2. **Negative norms.** Let G be a bounded domain in Euclidean n-space E^n with boundary ∂G of class[2] C^∞. Let $C^\infty(\overline{G})$ denote the set of all complex valued functions infinitely differentiable on $\overline{G}$, the closure of G. If $\mu=(\mu_1, \mu_2, \cdots, \mu_n)$ is any multi-index of length $|\mu|=\mu_1+\mu_2+\cdots+\mu_n$, we set

$$D^\mu = \partial^{|\mu|}/\partial x_1^{\mu_1}\partial x_2^{\mu_2}\cdots\partial x_n^{\mu_n}.$$

If s is any non-negative integer and p is any real number greater than one, we define

$$(2.1)\qquad \begin{aligned} \|u\|_{s,p} &= \left(\int_G \sum_{|\mu|\le s} |D^\mu u|^p dx\right)^{1/p}, \\ \|u\|_{-s,p} &= \underset{v\in C^\infty(\overline{G})}{\text{l.u.b.}} \frac{|(u,v)|}{\|v\|_{s,p'}}, \qquad p' = \frac{p}{p-1} \end{aligned}$$

for functions $u\in C^\infty(\overline{G})$, where $(u, v)=\int_G u\bar{v}dx$. Denote the completions of $C^\infty(\overline{G})$ with respect to these norms by $H^{s,p}(G)$ and $H^{-s,p}(G)$, respectively. They are obviously Banach spaces. Concerning them we state

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[2] Some assumptions are made for convenience only. Our results hold under less restrictive hypotheses.