NEW RESULTS AND OLD PROBLEMS IN FINITE TRANSFORMATION GROUPS[1]

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We shall be concerned with the topology of finite transformation groups on spaces of relatively simple character. This represents a rather small corner in the general theory, but the problems which one finds here seem to be of some interest and difficulty. By disposing of various low-dimensional cases, we shall try to show where the real difficulties begin.

1. **Definitions.** *A transformation group* (G, X) consists of a group G acting on a topological space X to form a group of homeomorphisms of X onto itself. It will be understood throughout this paper that G is finite. For a given transformation group or "action" (G, X) and subset $H \subset G$, we denote by $F(H; G, X)$ the fixed-point set of H—that is, the points x such that $hx = x$ for $h \in H$. We may of course denote this set simply by $F(H)$ when only one action is being considered. An action (G, X) is *g-free* if $F(g) = \emptyset$, *free* if it is g-free when $g \neq 1$, and *semi-free* if $F(G) = \emptyset$. Let X^r be the union of the fixed-point sets $F(g)$, $g \neq 1$, and let $X^f = X - X^r$. Since $gF(h) = F(ghg^{-1})$, the closed set X^r is invariant under the transformations $x \to gx$. Hence G acts on X^r (if X^r is not empty) hence also on X^f. The action (G, X^f) is free but the action (G, X^r) is not free. We call X^f the *free part* of X and X^r the *restricted part*; (G, X^r) may be called the restricted part of the action (G, X). An action (G, X) is *effective* if $F(g) \neq X$ when $g \neq 1$. In a given action, the set N of elements g with $F(g) = X$ is a normal subgroup of G and there is induced an action $(G/N, X)$ which is effective.

The sets Gx, $x \in X$, are the *orbits* of (G, X). They form a decomposition of X and the corresponding decomposition space, called the *orbit space* of the action, is denoted by X/G. The *stability group* G_x of x consists of all g such that $gx = x$.

An action (G, X) is of *class* C^k if X is a manifold of class C^k and the functions $x \to gx$ are of class C^k. When $k = 0$ we shall drop the manifold condition on X; every action is then of class C^0. A *differentiable* action is a C^1-action. (G, X) is *orthogonal* if X is a euclidean sphere or an open submanifold of a euclidean space and the trans-

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