ON THE MEAN ERGODIC THEOREM FOR SUBSEQUENCES

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1. Let Ω be a set, $\mathcal{A}$ a σ-algebra of subsets of Ω, and P a probability measure defined on $\mathcal{A}$, i.e. P is a nonnegative completely additive set function defined on $\mathcal{A}$, with $P(\Omega)=1$. If p is a number with $1 \leqq p < \infty$ we shall denote as is usual the Banach space of measurable functions f for which

$$\int_\Omega |f|^p \, dP < \infty \qquad \text{by } L_p(\Omega, \mathcal{A}, P).$$

Let T be a transformation mapping Ω onto Ω which is 1-1 and bimeasurable, i.e. if $A \in \mathcal{A}$ then

$$TA = \{y \mid y = Tx \text{ for some } x \in A\} \in \mathcal{A}$$

and

$$T^{-1}A = \{y \mid Ty \in A\} \in \mathcal{A}.$$

With these assumptions we have T^n defined for every integer n as a 1-1, onto, bimeasurable transformation. Henceforth we shall assume that every set considered is measurable, i.e. an element of $\mathcal{A}$. We shall say that P is invariant if $P(A)=P(TA)$ for every set A, P is ergodic if P is invariant and if $P(\bigcup_{n=-\infty}^{\infty} T^nA)=1$ for every set A for which $P(A)>0$, and finally P is strongly mixing if P is invariant and

$$\lim_n P[(T^nA) \cap B] = P(A)P(B)$$

for every pair of sets A, B. It is a consequence of the individual ergodic theorem that if P is invariant then P is ergodic if and only if

$$\lim_n \frac{1}{n} \sum_{i=0}^{n-1} \mathrm{P}[(T^iA) \cap B] = P(A)P(B)$$

for every pair of sets A, B. Thus every strongly mixing measure is ergodic.

It is the object of this paper to show that the strong mixing condition is equivalent to the validity of the mean ergodic theorem for every L_p space and every subsequence $\{T^{k_n}\}$ of the sequence $\{T^n\}$. More precisely let p be a number with $1 \leqq p < \infty$, let $f \in L_p(\Omega, \mathcal{A}, P)$