# CONTINUOUS FLOWS WITH CLOSED ORBITS[1]

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**Introduction.** A *continuous one-dimensional flow* (hereinafter called simply a *flow*) in a topological space $X$ is a continuous mapping from $R\times X$ onto $X$ ($R$ denotes the real line) which has the group property:

$$\phi(a, \phi(b, x)) = \phi(a + b, x), \qquad \text{for all } a, b \in R, x \in X.$$

Each point has an *orbit* $\vartheta(x) = \{\phi(a, x), a \in R\}$. We are especially interested in the set of points whose orbits consist of single points, that is, those fixed under the flow:

$$\phi(a, x) = x, \qquad \text{all } a \in R.$$

This set of fixed points is called *the invariant set of* $\phi$, and in this note, we announce a topological characterization of those sets which are the invariant sets of flows in the Euclidean plane all of whose orbits are closed.

1. We first turn our attention to the flows with compact orbits. Since an orbit can only be a point, a simple closed curve, or a simple arc without its endpoints, this hypothesis restricts us to flows whose orbits are points (the invariant set) and simple closed curves. It is easily seen that the invariant set is closed, and as a result, the intuition fixes at once on the conjecture that the points not invariant under $\phi$ constitute a family (necessarily countable) of disjoint open annuli (an annulus here will mean a homeomorph of the open set of points between two concentric circles). This latter takes some proof, mostly attention to details, but a few lemmas (whose content is visually obvious) suffice to give us

THEOREM 1. *If $\phi$ is a continuous flow in the (Euclidean) plane and if all the orbits of $\phi$ are compact, then the invariant set of $\phi$ is the complement of a (countable) disjoint union of open annuli.*

We are interested in the converse of this theorem. It is not hard to see that given any annulus, it is possible to construct a flow which leaves no point of the annulus fixed, has compact orbits, and extends continuously to the boundary where the flow has fixed points on the whole boundary. In fact, this can even be accomplished under the

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