THE WEAK HAUPTVERMUTUNG FOR CELLS AND SPHERES

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THEOREM. *If P and Q are two triangulations of the n-sphere (closed n-cell), there is a third triangulation M which can be obtained from either by subdivision. In fact, M can be obtained from either P or Q by subdivision of a single n-simplex.*

The following result, obtained recently by M. Brown [1], is the principal tool of both proofs.

LEMMA. *Let S^{n-1} be an $n-1$ sphere embedded in the n-sphere S^n. If S^{n-1} has a neighborhood in S^n homeomorphic to $S^{n-1}\times[-1, 1]$, in which S^{n-1} is embedded as $S^{n-1}\times 0$, then the closures of the complementary domains of S^{n-1} in S^n are both closed n-cells.*

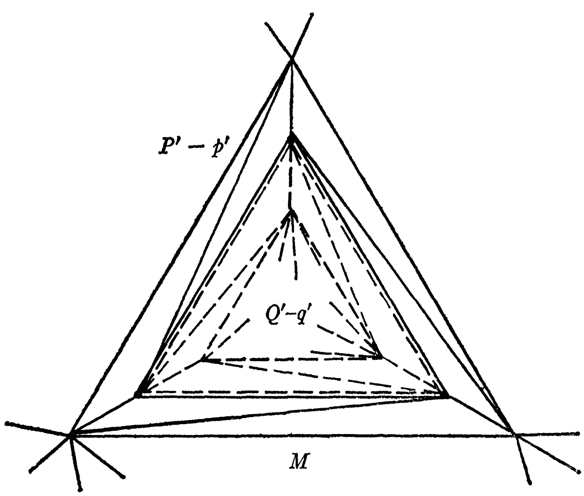

FIG. 1

We prove the theorem first for the n-sphere. Let p be an n-simplex of P, q an n-simplex of Q. Let p' be a smaller, concentric n-simplex inside p, and let P' be obtained from P by drawing p' inside p and triangulating the region ($S^{n-1}\times[0, 1]$) between the boundaries of p and p'. Similarly for q' and Q'. The boundaries of $|p'|$ and $|q'|$ have neighborhoods as required in the lemma, so they split $|P'|$, resp. $|Q'|$, into two closed n-cells, one of which is $|p'|$, resp. $|q'|$, and the