RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

SOLUTION OF THE EQUATION $(pz+q)e^z=rz+s$

BY E. M. WRIGHT

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An accurate knowledge of the roots of the equation in the title plays a part in the solution and application of linear difference-differential equations and in other theories. If $pr=0$, it is easy to transform our equation into $ze^z=a$, an equation dealt with in [1]. If $pr\neq 0$ and p, q, r, s are real, our equation transforms into one or other of the equations

$$(z+b)e^{z+a} = -(z-b) \tag{1}$$

and

$$(z+b)e^{z+a} = z-b, \tag{2}$$

where a, b are real.

We write $z=x+iy$, $w=u+iv$, where x, y, u, v are real. We cut the z-plane along the real axis from $z=-b$ to $z=b$ and write

$$w = z - \log\{(z-b)/(z+b)\}, \tag{3}$$

taking the logarithm real on the real axis outwith the cut, and writing $z=z(w)$ for the inverse function defined by (3). Clearly

$$\frac{dw}{dz} = \frac{z^2-b^2-2b}{z^2-b^2}$$

vanishes when $z=\zeta$, where $\zeta^2=b^2+2b$. We write

$$\omega = w(\zeta) = \zeta - \log\{(\zeta-b)/(\zeta+b)\} = \zeta + \log(b+1+\zeta).$$

If $b>0$, we have $\omega=\pm(b^2+2b)^{1/2}+\log\{b+1+(b^2+2b)^{1/2}\}$. If $-2<b<0$ and we write $b+1=\cos\theta$, $0<\theta<\pi$, we have $\zeta=\pm i\sin\theta$, $\omega=\pm i(\theta+\sin\theta)$, so that $0<|\omega|<\pi$. If $b\leqq -2$, we have $\omega=\pm\{(b^2+2b)^{1/2}+\log(|b+1|-(b^2+2b)^{1/2})\}\pm\pi i$. If $b=-2$, we have $\zeta=0$ and $\omega=\pm i\pi$. We observe that, for any fixed b, there may be two or four values of ω, but all have the same modulus.

The inverse function $z(w)$ is uniquely defined if either (i) $|w|>|\omega|$