Analytic function theory, vol. 1. By Einar Hille, Boston, Ginn and Co., 1959. 11+308 pp. $6.50.

This is the first volume of a two volume work on analytic functions. It is an excellent book having several features which should make it attractive to the teacher and the student.

The first is the appearance of historical footnotes containing information about important theorems and their discoverers. One learns for example that the revival of Italian mathematics is conventionally dated from the journey which Enrico Betti, Francesco Brioschi and Felice Casorati made to France and Germany in 1858. Another feature is the skillful threading of ideas through the book. One first encounters an idea in an exercise, later it appears in an illustration, then it receives specific notice in a remark or discussion and finally is formalized in the statement of a definition or theorem.

This type of development is well illustrated by the treatment of inverse functions which originates in Chapter 3 and culminates in the last section of the book with a derivation of the properties of the k roots near a zero or order k. The end of volume 1 finds many of these ideas still developing, for example that of analytic continuation. Perhaps some will not reach maturity. Singular integrals may fall in this category. In any event the student will not fail to find many directions in which to continue his studies, aided by references at the end of each chapter. A contrary objective is fulfilled by the exercises at the end of each section which are designed primarily to illustrate the material in the section.

While the author emphasizes the historical origins of the subject the reader is also introduced to more recent points of view through the description of the algebraic structure of the various function classes as they arise. Another conspicuous merit is the elegance of detail and construction. One may point to the admissible functions introduced in the discussion of double series or the parallel curves used in the discussion of analytic continuation across a smooth arc.

As stated in the foreword the core of this volume consists of Chapters 4, 5, 7, 8, and 9. The remaining material in four chapters and three appendices is either introductory or supplementary. Chapters 1, 2 and 3 introduce the real and complex numbers, the topology of the complex plane and Möbius transformations. The section on Möbius transformations comprises only twelve pages yet contains discussions of the important subgroups: rigid rotations of the sphere, transformations leaving the unit circle invariant and transformations leaving two points fixed. Parabolic, elliptic, hyperbolic and loxodromic transformations are described. A proof of the Jordan theorem for polygons is contained in an appendix. Several threads of ideas