A CERTAIN CLASS OF TOPOLOGICAL PROPERTIES[1]

R. L. WILDER[2]

I. **Introduction.** About thirty years ago it was considered that Topology had two aspects, according to which its practitioners could be roughly divided into two schools. These aspects would today be termed *global* and *local*, and those whose interests lay mainly in global properties of a space or configuration formed one school, while those whose chief interest lay in local properties formed the other. The causes for this division of interests were methodological: The combinatorial method, stemming chiefly from the work of Poincaré and Veblen, yielded only global invariants such as the classical Betti numbers and duality theorems; while the set-theoretic method, starting from the strictly local notion of limit point, was particularly suited for the study of local properties. Thus a more accurate differentiation between the schools could be made according to method; for while the set-theoretic method allowed of the study of such concepts as local connectedness, the concepts of connectedness and unicoherence, for example, were global; and soon the combinatorial method became adapted to the study of local properties.[3]

While this division into schools has essentially disappeared (although it is still convenient to distinguish the set-theoretic and algebraic methods), the distinction between global and local aspects of topological spaces furnishes a useful theoretical perspective. It is the purpose of the present discussion to focus attention on properties which in general lie between the global and local, and which might therefore be termed *medial*.[4] For example, the property of every open subset of a space being a G_δ is medial, in that it holds for all open sets, large and small.

As in the case of such local properties as that of local connectedness, however, we find it more fruitful, in defining medial properties, to use *pairs* of open sets rather than single open sets. The original definition of local connectedness at a point x stipulated that for arbitrary open set U containing x there exists an open set V such that

[1] Presidential Address delivered before the Annual Meeting of the Society in Cincinnati on January 29, 1958; received by the editors September 4, 1959.

[2] Aid in the preparation of this address was received from National Science Foundation Grants G-2783 and G-5612.

[3] For a more extended discussion of the situation thirty years ago, see my symposium lecture [9].

[4] We resist the temptation to introduce the barbarism "glocal," on grounds of its ease of confusion with "global."