ON RANDOM SOLUTIONS OF FREDHOLM INTEGRAL EQUATIONS

BY A. T. BHARUCHA-REID[1]

Communicated by Richard Bellman, October 27, 1959

1. **Introduction.** In this note we consider the solution of the *random operator equation*

$$(T(\omega) - \lambda I)x(u, \omega) = y(u, \omega) \tag{1}$$

when the operator $T(\omega)$ is a random integral operator of the Fredholm type on a concrete Banach space of generalized random variables. In [**3**] (the proofs will appear in [**4**]) we announced some results in the theory of random operator equations; and we refer to the above paper, or to [**6**] for all definitions. We begin our study of concrete random operator equations with the Fedholm integral equations for the following reasons: (1) there is a well-developed theory of these equations (cf. [**11**] and [**13**]), (2) the relationship between these equations and Volterra integral equations, algebraic systems of linear equations, and Sturm-Liouville systems of differential equations, and (3) the widespread occurrence of these equations in mathematical physics and other branches of applied mathematics.

2. **The stochastic boundary value problem for integral equations.** In general, the classical (i.e., deterministic or nonstochastic) boundary value problem can be described as follows: Given a functional equation

$$\mathcal{P}(x(u)) = 0 \tag{2}$$

defined on a domain U, with boundary F, in k-dimensional Euclidean, space R_k, find a function $x(u)$ satisfying Equation (2) in U and taking prescribed values on the boundary F; that is $\lim_{u \to u_0 \in F} x(u) = \gamma(u_0)$. The functional $\mathcal{P}$ in (2) is a mapping of the abstract space of function $x(u)$ onto itself.

However, in many boundary value problems arising in various field of applications, the boundary conditions cannot be expressed by a single well-determined or known function $\gamma(u_0)$; hence it is necessary to consider this problem within the framework of probability theory, and consider a collection or set of functions, say $\Gamma = \{\gamma(u, \omega), \omega \in \Omega\}$,

[1] This research was supported in part by funds provided under Contract No. DA-04-200-ORD-651 with the Office of Ordnance Research, U. S. Army.