CLOSED IDEALS IN GROUP ALGEBRAS

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Communicated October 26, 1959

Let $A(G)$ be the set of all Fourier transforms on the locally compact abelian group G, i.e., the set of all f of the form

$$f(x) = \int_{\Gamma} (x, \gamma)F(\gamma)d\gamma \qquad (x \in G,\ F \in L^1(\Gamma)),$$

where Γ is the dual group of G and (x, γ) is the value of the character γ at the point x. With the norm

$$\|f\| = \int_{\Gamma} |F(\gamma)| d\gamma$$

$A(G)$ is a commutative Banach algebra, and G is its maximal ideal space.

If I is a closed ideal in $A(G)$, let $Z(I)$ be the set of all $x \in G$ such that $f(x) = 0$ for every $f \in I$. Malliavin [**3**; **4**; **5**] has recently solved a problem of long standing by proving that in every nondiscrete G there is a closed set E such that $E = Z(I_1) = Z(I_2)$ for two *distinct* closed ideals I_1 and I_2 in $A(G)$. Combined with an older result of Helson [**1**] this implies that there are infinitely many closed ideals I in $A(G)$ with $Z(I) = E$.

It is the purpose of this note to point out that Malliavin's construction for compact G (he reduced the general case to this) yields an even more specific result:

THEOREM. *Suppose G is an infinite compact abelian group. There is a real $f \in A(G)$ such that the closed ideals I_n generated by the powers f^n $(n = 1, 2, 3, \cdots)$ are all distinct.*

We sketch the proof. If $g \in A(G)$ and u is a real number, we define $a_\gamma(u)$ by

$$(1) \qquad e^{iug(x)} = \sum_{\gamma \in \Gamma} a_\gamma(u) \cdot (x, \gamma) \qquad (x \in G).$$

Malliavin [**5**] constructed a real $g \in A(G)$ for which

$$(2) \qquad | a_\gamma(u) | < \exp(-C | u |^{1/2}) \qquad (\gamma \in \Gamma),$$

where $C > 0$ is independent of γ. (The exponent 1/2 in (2) could be

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