THE BOUNDARY OF A SIMPLY CONNECTED DOMAIN[1]

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1. **Prime ends.** The foundation for the study of boundaries of simply connected domains in the plane was laid by Carathéodory [2], who defined ends in general and prime ends in particular, and who classified prime ends into four kinds. To facilitate a brief survey of the subject of prime ends, I introduce a few definitions concerning a simply connected domain B. In my definitions, I follow essentially the work of Carathéodory, except that, for the sake of brevity, I omit the description of ends in general and aim directly at prime ends.

A sequence K of crosscuts c_n of B is a *chain* provided

(i) the diameter of c_n tends to 0 as $n \to \infty$;

(ii) for each index n, the set $\bar{c}_n \cap \bar{c}_{n+1}$ (where the bar indicates closure) is empty;

(iii) some fixed point O in B cannot be joined to any crosscut c_n $(n > 1)$ by any path in B which does not meet the crosscut c_{n-1}.

Two chains $K = \{c_n\}$ and $K' = \{c_n'\}$ in B are *equivalent* provided each crosscut c_n effects a separation, relative to B, of the point O from all except finitely many of the crosscuts c_n'.

An equivalence class of chains in B is a *prime end* of B.

If P is a prime end of B, let $K = \{c_n\}$ denote a chain which belongs to P, and for each index n let B_n denote that subdomain of B which is determined by c_n and does not contain the point O. The set $I(P) = \bigcap \bar{B}_n$ will be called the *impression* of P.

It should be remarked that Carathéodory and some other writers applied the term *prime end* to the point set $I(P)$, but that they regarded as distinct two prime ends P_1 and P_2 corresponding to two nonequivalent chains, even in cases where the two sets $I(P_1)$ and $I(P_2)$ are identical. The distinction between a prime end and its impression formalizes the ideas which are involved.

Carathéodory's principal theorem on the correspondence between boundaries under conformal mappings [2, p. 350] can be expressed as follows: *If $f(z)$ maps the unit disk conformally and one-to-one onto the*

An address delivered before the Evanston meeting of the Society on November 23, 1956, by invitation of the Committee to Select Hour Speakers for Western Sectional Meetings; received by the editors March 20, 1957.

[1] The address was prepared under Contract DA 20-018-ORD-13585 with the Office of Ordnance Research, U. S. Army.