STRUCTURE AND REPRESENTATION OF NONASSOCIATIVE ALGEBRAS[1]

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1. Introduction. By a nonassociative algebra is meant a vector space which is equipped with a bilinear multiplication. If the multiplication is associative, we have the familiar notion of an associative algebra. Also Lie algebras are an essential tool for the study of Lie groups, as is well known. However, we shall not be particularly concerned with associative or Lie algebras today, except as models of what well-behaved nonassociative algebras should be.

The study of algebras which are not associative is not a recent development. The 8-dimensional algebra of Cayley numbers was known as early as 1845 [**26**].[2] However, it is within the last fifteen years that the study has received its greatest impetus. With a few notable exceptions—and always excepting Lie algebras of course—there were only isolated results before that time. By now a pattern is emerging, for certain finite-dimensional algebras at least, and this paper is an exposition of some of the principal results achieved recently in the structure and representation of finite-dimensional nonassociative algebras.

A. A. Albert has been the prime mover in this study. The depth and scope of the results, at least for one class of nonassociative algebras, may be judged by the fact that N. Jacobson will give the Colloquium lectures at the Summer Meeting of the Society on the topic of Jordan algebras. Except for this reference to these two men, I shall not attempt to identify the authors of any theorems in this talk. The bibliography of the published paper will speak for itself.

2. The associative and Lie theories as models. Let F be an arbitrary field and A be a finite-dimensional associative algebra over F. It is well known that there is an ideal N, called the radical of A, which is the unique maximal nilideal of A (that is, the maximal ideal consisting entirely of nilpotent elements). Furthermore, N is nilpotent in the sense that there is an integer t with the property that any product $z_1z_2 \cdots z_t$ of t elements from N is zero; hence N is also the

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[2] Numbers in brackers refer to the bibliography at the end of the paper.