THE CONVOLUTION TRANSFORM

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Introduction. The material I am reporting on here was prepared in collaboration with I. I. Hirschman. It will presently appear in book form in the Princeton Mathematical Series. I wish also to refer at once to the researches of I. J. Schoenberg and his students. Their work has been closely related to ours and has supplemented it in certain respects. Let me call attention especially to an article of Schoenberg [5, p. 199] in this Bulletin where the whole field is outlined and the historical development is traced. In view of the existence of this paper I shall try to avoid any parallel development here. Let me take rather a heuristic point of view and concentrate chiefly on trying to entertain you with what seems to me a fascinating subject.

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1. **Convolutions.** Perhaps the most familiar use of the operation of convolution occurs in its application to one-sided sequences $\{a_n\}_0^\infty$, $\{b_n\}_0^\infty$. The convolution (Faltung) of these two sequences is defined as the new sequence $\{c_n\}_0^\infty$,

$$c_n = \sum_{k=0}^{n} a_k b_{n-k} = \sum_{k=0}^{n} a_{n-k} b_k. \tag{1.1}$$

The operation arises when power series are multiplied together:

$$\sum_{k=0}^{\infty} a_k z^k \sum_{k=0}^{\infty} b_k z^k = \sum_{k=0}^{\infty} c_k z^k.$$

The convolution of two-sided sequences,

$$c_n = \sum_{k=-\infty}^{\infty} a_k b_{n-k} = \sum_{k=-\infty}^{\infty} a_{n-k} b_k, \tag{1.2}$$

presents itself when two Laurent series are multiplied.

Hardly less familiar is the continuous analogue of (1.2),

$$c(x) = \int_{-\infty}^{\infty} a(x-y)b(y)dy = \int_{-\infty}^{\infty} a(y)b(x-y)dy, \tag{1.3}$$

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