APPROXIMATE SOLUTIONS OF FREDHOLM-TYPE INTEGRAL EQUATIONS

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1. **Introduction.** The integral equation whose approximate solution will be discussed here is the linear nonhomogeneous equation of Fredholm type and second kind

$$x(s) - \int_0^1 K(s, t)x(t)dt = y(s), \qquad 0 \leqq s \leqq 1, \tag{1.1}$$

where the functions $K(s, t)$—the "kernel"—and $y(s)$ are known. The question posed is this: Given an approximate method for solving (1.1), how can one estimate the error?

Since late in the nineteenth century the importance of (1.1) in mathematical physics has been recognized, along with that of

$$x(s) - \lambda \int_0^1 K(s, t)x(t)dt = 0, \qquad 0 \leqq s \leqq 1, \tag{1.2}$$

in which one is to determine values of λ (proper values) such that a continuous solution $x(s) \not\equiv 0$ exists. Typical problems leading to equations like (1.1) are the Dirichlet and Neumann problems of potential theory; to (1.2), time-dependent problems in elastic vibration and heat flow, by "separating out" the time.

It is now fifty years since Fredholm published his distinguished paper in Acta Mathematica [**10**],[2] in which he gave the first detailed account of the existence and multiplicity of solutions of (1.1) and (1.2). Few mathematical publications have stimulated so much further work. Several papers appeared in which physical problems were set up in terms of integral equations (elasticity, gas theory, etc.); some were on numerical solutions; most of the many theoretical papers followed the now familiar trend toward greater generality and abstraction. Hilbert recognized analogies with Euclidean geometry, except that now space was infinite-dimensional [**13**]; through Fréchet and F. Riesz this led to Banach spaces and more rarefied

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[2] Numbers in brackets refer to the bibliography at the end of the paper.