RESEARCH PROBLEMS

8. Andrew Sobczyk: *Projections in Banach spaces.*

Does there exist any infinitely-dimensional, separable, closed linear subspace X of the nonseparable space (m) of all bounded sequences, such that there is a continuous projection of (m) onto X? Phillips and Sobczyk have shown that there is no continuous projection of (m) onto (c_0), the subspace of all sequences convergent to zero. Sobczyk has shown that for any separable closed linear subspace W, $W \supset (c_0)$, there is a projection of bound 2 of W onto (c_0), and therefore no continuous projection of (m) onto W. A lemma of Murray states that there is a continuous projection onto a closed linear subspace Y if and only if there is a complementary closed linear subspace Z. For any Banach space $U \supset (m)$, there is a projection of bound 1 onto (m). Does there exist a pair of complementary closed linear subspaces Y, Z for (m), such that neither Y nor Z is separable or isomorphic with (m)? Similar questions may be asked concerning the existence of closed projections. References: D. B. Goodner, Trans. Amer. Math. Soc. vol. 69 (1950) pp. 89–108. F. J. Murray, Bull. Amer. Math. Soc. vol. 48 (1942) pp. 76–93. R. S. Phillips, Trans. Amer. Math. Soc. vol. 48 (1940) pp. 516–541. A. Sobczyk, Bull. Amer. Math. Soc. vol. 47 (1941) pp. 938–947; Duke Math. J. vol. 8 (1941) pp. 78–106; Trans. Amer. Math. Soc. vol. 55 (1944) pp. 153–169. (Received February 11, 1954.)

9. Lowell J. Paige: *Elements of odd order in a finite group.*

Let G be a finite group of order $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k} \cdot 2^s$, where $p_1, p_2, \cdots, p_k$ are distinct odd primes. Let P be a Sylow 2-subgroup of G and let S be the set of all elements of G satisfying the equation $x^r = 1$, where $r = p_1^{\alpha_1} \cdots p_k^{\alpha_k}$. For the coset expansion of G by P,

$$G = g_1 P + g_2 P + \cdots + g_r P,$$

is there an element of S in each coset $\{g_i P\}$ $(i = 1, 2, \cdots, r)$? Note that the problem is trivial if P is normal or if P is its own normalizer in G and the intersection of P with each of its conjugates is the identity. (Received February 15, 1954.)

10. Casper Goffman: *The group of similarity transformations of a simply ordered set.*

Let S be a simply ordered set. A similarity transformation of S is a one-to-one correspondence between S and itself which preserves order. The similarity transformations of S form a group $G(S)$. For a well ordered set S, $G(S)$ consists of a single element, but there are other ordered sets with this property. The problem is the following: (a) characterize the ordered sets S for which $G(S)$ consists of one element; (b) characterize the ordered sets S for which $G(S)$ is abelian; (c) characterize the groups G for which there is an ordered set S such that $G = G(S)$. (Received February 17, 1954.)

11. O. Taussky: *Multiply monotonic sequences.*

Szegö (Duke Math. J. vol. 8 (1941) pp. 559–564) investigated the following theorem of Fejér: Let the sequence $\{a_n\}$ be monotonic of order 4. Then the power series $f(z) = \sum_{n=0}^{\infty} a_n z^n$ is regular and univalent for $|z| < 1$. Szegö proved that this theorem remains true for monotonic sequences of order 3, but is not true for monotonic se-