self-contained, while on the other hand the student who wishes to pursue the subject further can do so.

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Bernstein polynomials. By G. G. Lorentz. (Mathematical Expositions, no. 8.) University of Toronto Press, 1953. 10+130 pp. $5.75.

For a function $f(x)$ defined for $0 \leqq x \leqq 1$ the formula

$$B_n(x) = \sum_{\nu=0}^{n} f\left(\frac{\nu}{n}\right)\binom{n}{\nu} x^\nu (1 - x)^{n-\nu}$$

defines the Bernstein polynomial of order n of $f(x)$. Evidently $B_n(x)$ is a polynomial in x of order less than or equal to n. S. Bernstein, who introduced these polynomials, proved that if $f(x)$ is continuous for $0 \leqq x \leqq 1$, then

$$\lim_{n \to \infty} B_n(x) = f(x)$$

uniformly for $0 \leqq x \leqq 1$, thus obtaining a particularly simple and elegant proof of the Weierstrass approximation theorem. Bernstein polynomials are of interest in themselves, and in addition play an important role in several other mathematical topics, notably in the finite moment problem and in the related theory of Hausdorff summability.

Chapter I deals with the approximation of continuous functions by Bernstein polynomials. A great many results are proved of which the following may serve as an example: if $f(x)$ satisfies a Lipschitz condition of order α, $0 < \alpha < 1$, then

$$\left| f(x) - B_n(x) \right| = O(n^{-\alpha/2}) \qquad (n \to \infty),$$

uniformly for $0 \leqq x \leqq 1$. Care is taken to explain the relation of these results to the general theory of polynomial approximation.

Chapter II treats the changes which are necessary if $f(x)$ is no longer continuous, but merely integrable, or even only measurable. For example if $f(x)$ is integrable $B_n(x)$ may be replaced by

$$P_n(x) = \sum_{\nu=0}^{n} \binom{n}{\nu} x^\nu (1 - x)^{n-\nu} \left[(n + 1) \int_{\nu/(n+1)}^{(\nu+1)/(n+1)} f(t)dt \right].$$

It is shown that

$$\lim_{n \to \infty} P_n(x) = f(x)$$