SOME NEW ALGEBRAIC METHODS IN TOPOLOGY

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1. **Introduction.** The purpose of this address is solely expository. It is intended to give the mathematician who is not an expert in algebraic topology a picture of some of the newer algebraic techniques and machinery which have recently become common in that subject. The expert is warned that this exposition contains no methods or results which have not already been published.

We shall concentrate attention on the spectral sequence, a topic initiated by Leray. By its use, one can investigate the homology structure of a fibre space in terms of the base space and fibre. This method has been applied with considerable success to the study of the topological structure of Lie groups and homogeneous manifolds. Other important applications have been made in the subject of differential geometry in the large.

2. **Graded groups.** In algebraic topology, one associates with each topological space certain algebraic structures, such as groups, rings, vector spaces, modules, etc. In this address, we shall for the sake of simplicity restrict our attention mainly to certain abelian groups which are associated with topological spaces. Almost everything we shall say could be equally well applied to the case of vector spaces over a given field, or more generally, to modules over a given commutative ring. And with a little additional effort, we could consider the various rings that are associated with a space.

Usually it turns out that one associates with a topological space X not a single abelian group, but a whole sequence of abelian groups. The most important examples are the following:

The n-dimensional homology group of X with coefficients in an arbitrary abelian group G, denoted by $H_n(X, G)$ $(n=0, 1, 2, \cdots)$.

The n-dimensional cohomology group of X with coefficients in an arbitrary abelian group G, denoted by $H^n(X, G)$ $(n=0, 1, 2, \cdots)$.

The n-dimensional homotopy group of X, denoted by $\pi_n(X)$ $(n=1, 2, 3, \cdots)$. These groups were introduced by Hurewicz in 1935. $\pi_1(X)$ is the ordinary fundamental group, which need not be abelian. However, for $n>1$, $\pi_n(X)$ is abelian.

These groups are the very basis of algebraic topology. Many im-

An address delivered before the New York meeting of the Society on April 24, 1953 by invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings; received by the editors June 1, 1953.