Altogether we have here a very fine addition to mathematical literature.

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*On the metamathematics of algebra.* By A. Robinson. (Studies in Logic and the Foundations of Mathematics.) Amsterdam, North-Holland, 1951. 9+195 pp. 18.00 fl.

The purpose of this book is to show how the methods of symbolic logic may be applied to derive new results in algebra. This is a somewhat novel idea, since usually symbolic logic has been used to strengthen the foundations, rather than to add to the superstructure, of mathematical systems. The results obtained are in the nature of metatheorems rather than theorems; they deal with entire classes of algebraic systems rather than with one system at a time.

Some typical results are the following: 1. Any theorem formulated in the restricted calculus of predicates (in terms of equality, addition, and multiplication) which is true for all commutative fields of characteristic 0, is true for all commutative fields of characteristic $p > p_0$, where $p_0$ is a constant depending on the theorem. 2. Any theorem of the restricted calculus of predicates which is true for all non-Archimedean ordered fields is true for all ordered fields. 3. Any theorem of the restricted calculus of predicates which is true for the field of all algebraic numbers is true for any other algebraically closed field of characteristic 0.

A feature of the method is the following: since the results require only that the algebraic systems dealt with have certain broad, general properties, they naturally suggest significant generalizations of various algebraic notions. Examples of concepts generalized are: that of algebraic number, or a number algebraic with respect to a given commutative field; the notion of the polynomial ring obtained by adjoining $n$ indeterminates to a commutative ring; and the concept of ideal. In general, the notions that may be handled by the author's method are those capable of being formulated within the restricted predicate calculus.

A number of the results obtained are significant for symbolic logic as well as for algebra. For example, it is shown that every model of an axiomatic system formulated in the restricted calculus of predicates can be extended. Consequently such a model cannot satisfy an axiom of completeness in Hilbert's sense. Since the concept of an ordered field can be formalized within the restricted calculus of predicates while the concept of an Archimedean field does in fact possess a model which is complete in Hilbert's sense (namely the field of all