SOME ASPECTS OF THE SEQUENTIAL DESIGN OF EXPERIMENTS

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1. Introduction. Until recently, statistical theory has been restricted to the design and analysis of sampling experiments in which the size and composition of the samples are completely determined before the experimentation begins. The reasons for this are partly historical, dating back to the time when the statistician was consulted, if at all, only after the experiment was over, and partly intrinsic in the mathematical difficulty of working with anything but a fixed number of independent random variables. A major advance now appears to be in the making with the creation of a theory of the *sequential design* of experiments, in which the size and composition of the samples are not fixed in advance but are functions of the observations themselves.

The first important departure from fixed sample size came in the field of industrial quality control, with the double sampling inspection method of Dodge and Romig [1]. Here there is only one population to be sampled, and the question at issue is whether the proportion of defectives in a lot exceeds a given level. A preliminary sample of n_1 objects is drawn from the lot and the number x of defectives noted. If x is less than a fixed value a the lot is accepted without further sampling, if x is greater than a fixed value b $(a<b)$ the lot is rejected without further sampling, but if $a \leqq x \leqq b$ then a second sample, of size n_2, is drawn, and the decision to accept or reject the lot is made on the basis of the number of defectives in the total sample of n_1+n_2 objects. The total sample size n is thus a random variable with two values, n_1 and n_1+n_2, and the value of n is stochastically dependent on the observations. A logical extension of the idea of double sampling came during World War II with the development, chiefly by Wald, of sequential analysis [2], in which the observations are made one by one and the decision to terminate sampling and to accept or reject the lot (or, more generally, to accept or reject whatever statistical "null hypothesis" is being tested) can come at any stage. The total sample size n now becomes a random variable capable in principle of assuming infinitely many values, although in practice a finite upper limit on n is usually set. The advantage of sequential

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