

Alternatives (1) and (2) represent extremes of optimism and pessimism respectively concerning the statistician's attitude toward Nature. Most statisticians will probably find themselves somewhere between the two extremes, and faced with the problem of how to utilize their rather vague feelings about the frequency with which various possible  $F$ 's can be expected to occur. For this reason, this reviewer suspects that minimax solutions as such are likely to be of little interest in statistics. For example, on p. 142 Wald cites the minimax point estimate of the mean  $\theta$  of a binomial variate; the corresponding risk function is a constant  $r_0 = [2(1 + N^{1/2})]^{-2}$ . The traditional (non-minimax) estimate has risk function  $\theta(1 - \theta)/N$ . For large  $N$  the ratio of this to  $r_0$  is near zero except in a small interval about  $\theta = 1/2$ , where it is slightly greater than 1. It is very hard to believe in the superiority of the minimax estimate in this case, which is by no means unusual in its nature.

To those who are indifferent to minimax solutions the principal interest of the book will lie in the main theorem that, under very general conditions, the class  $\mathcal{B}$  of all Bayes solutions is *essentially complete* in the following sense: for any decision function  $\delta$  there exists a  $\delta^*$  in  $\mathcal{B}$  such that  $r(F, \delta^*) \leq r(F, \delta)$  for all  $F$  in  $\Omega$ . (Mathematically, this theorem represents a highly nontrivial extension of the method of Lagrange multipliers in the calculus of variations.) There is obviously no loss involved in restricting the choice of a decision function to any essentially complete class, in particular to  $\mathcal{B}$ . But even a minimal essentially complete class will usually be so large that further reduction is necessary before the statistician can turn the problem of selecting a decision rule over to the experimenter. One criterion for reduction, the minimax principle, has already been mentioned. Other criteria exist (unbiasedness, invariance, and so on) but are not dealt with in the present volume.

The book makes effective use of the modern theory of measure and integration, and operates at a high level of rigor and abstraction. For this reason few statisticians will be prepared to read it, yet its ultimate liberating effect on statistical theory will be great. It is to be hoped that so rich and stimulating a book as this will reach an audience among mathematicians.

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*Introduction to the theory of algebraic functions of one variable.* By C. Chevalley. (Mathematical Surveys, no. 6.) New York, American Mathematical Society, 1951. 12+188 pp. \$4.00.

Here is algebra with a vengeance; algebraic austerity could go no