SOME QUESTIONS CONCERNING ALTERNATIVE RINGS

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1. **Introduction.** Our purpose is to summarize our present knowledge of alternative rings in the case in which no a priori finiteness assumptions are made and to indicate a number of problems in this field. One defines an alternative ring by replacing the law $a(bc) = (ab)c$ in the definition of an associative ring by the laws $a(ab) = a^2b$ and $(ab)b = ab^2$. The name is derived from the fact that the *associator* $(a, b, c) = (ab)c - a(bc)$ is an alternating function of its arguments. The name as well as much of our knowledge of the finite-dimensional case is due to M. Zorn [**52–55**],[1] although N. Jacobson [**25**], A. A. Albert [**1**], R. D. Schafer [**41–43**], and Dubisch and Perlis [**19**] have also contributed.

We shall use the terms *ring* and *algebra* in place of *nonassociative ring* and *nonassociative algebra*. If the nonzero elements of a ring form a *loop* [**4**] under multiplication (that is, if each pair of elements in the equation $ab = c$ uniquely determines the remaining element and a unit element 1 is present), we call the ring a *division ring* [cf. **2**; **16**]. The *center* [**2**; **26**] of a ring A consists of those elements c in A for which $cx = xc$ and $(cx)y = c(xy) = x(cy)$ for every x and y in A. When the center of A is a division ring, then A is a vector space over its center and we call the dimension of this vector space the *dimension* of A.

We divide our discussion into three parts wherein the primary interest is geometric, algebraic, and topological, respectively.

2. **Geometry.** Ruth Moufang [**33–38**] was the first to derive the geometric meaning of the alternative law as a weak form of Desargues' Theorem in plane projective geometry. Marshall Hall, Jr. [**22–23**] has given a new proof which is mainly algebraic and which avoids assumptions concerning order or characteristic in the plane. In its affine form, the theorem used by Hall merely asserts that *if corresponding vertices of two triangles are on parallel lines, while two pairs of corresponding sides are parallel, then the remaining sides are also parallel.* The works of Moufang and of Hall suggest the following questions.

1. Is there an ordered alternative division ring which is not associative? (Moufang [**38**])

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[1] Numbers in brackets denote references given at the end of the paper.