COMPLEXES AND HOMOTOPY CHAINS

KURT REIDEMEISTER

The theory I have to speak about is a chapter of the algebraic topology of complexes. Its definition parallels the classical homology theory.

Let A be a complex with the oriented cells a_i^k, where k is the dimension number. Then the homology theory starts with the free Abelian groups of chains

$$c^k = \sum \xi a^k, \qquad \xi \in N,\ a^k \in A,$$

generated by the a_i^k (N is the set of the integers) and the boundary homomorphism of chains

$$(c^k)^\cdot = \sum \xi (a^k)^\cdot$$

where

$$(a_i^k)^\cdot = \sum \rho_{ij}^k a_j^{k-1}$$

is the boundary chain of the oriented cell a_i^k, the ρ_{ij}^k being the incidence numbers of the cells a_i^k, a_j^{k-1}. These chains and boundary matrices change by subdivision of the complex A, and the homology groups are the invariants with regard to this process.

The chains and boundary matrices which I introduce are defined for complexes U with an adjoined group G of mappings γ of U in itself, that is, of mappings

$$\gamma u^k = \bar{u}^k$$

of the cells u^k of U preserving the dimension, the orientation, and the incidence relations of cells. The subdivision of the euclidean plane in squares, which is mapped in itself by the group of translations with integer coefficients, is an example for a complex U.

The mappings γ of the cells induce automorphisms γ of the chains of U,

$$\gamma c^k = \gamma \sum \xi u^k = \sum \xi (\gamma u^k),$$

and these automorphisms commute with the boundary homomorphism; for

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