L-S-HOMOTOPY CLASSES ON THE TOPOLOGICAL IMAGE OF A PROJECTIVE PLANE

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1. Introduction. Models for the L-S-(locally simple) homotopy classes of closed $p$-curves ($p$ = parameterized) on any 2-manifold $S$ have been announced in Morse [1]. Proofs have been given only for the case in which $S$ is orientable. The present paper will treat the case in which $S$ is the top. (topological) image of a projective plane. The proofs in the case of a general non-orientable surface can be given by an appropriate modification of methods of Morse [1] and of the present paper.

Recall that one writes $f \simeq 0$ when $f$ is a closed $p$-curve homotopic to zero. Deferring technical definitions until later sections, we can state the principal theorem as follows.

**Theorem 1.1.** Let $h$ be a simple closed $p$-curve on the top. image $S$ of a projective plane with $h \not\simeq 0$ on $S$. Let $h^{(n)}$ ($n > 0$) be a closed $p$-curve on $S$ which traces $h$ $n$ times. Any L-S-closed $p$-curve $f$ on $S$ is in the L-S-homotopy class of $h^{(1)}$ or $h^{(3)}$ if $h \not\simeq 0$, and of $h^{(2)}$ or $h^{(4)}$ if $h \simeq 0$. No two of the $p$-curves $h^{(1)}, h^{(2)}, h^{(3)}, h^{(4)}$ are in the same L-S-homotopy class.

For theorems on regular closed curves in the plane see Whitney, and H. Hopf. For L-S-closed curves in the plane see Morse [2] and Morse and Heins [1]. For a use of L-S-curves in studying deformation classes of meromorphic functions see Morse and Heins [2].

2. L-S-curves and deformations. Let $C$ represent the unit circle on which $|z| = 1$ in the plane of the complex variable $z = u + iv$. With $z = e^{i\theta}$ on $C$, we assign $C$ the sense of increasing $\theta$. Let $S$ be an arbitrary 2-manifold. A closed $p$-curve on $S$ is a continuous mapping $f$ of $C$ into $S$ such that the image of $z$ in $C$ is a point $f(z)$ in $S$. Two $p$-curves $f_1$ and $f_2$ are regarded as the same if and only if

$$f_1(z) = f_2(z)$$

for every $z$ in $C$. The union of the points $f(z)$ in $S$ as $z$ ranges over $C$ is called the carrier of $f$. The simplest case arises when the points $f(z)$ are in 1-1 correspondence with their antecedents $z$ in $C$, and in this case $f$ is termed simple.

Let $f$ be a continuous mapping of $C$ into $S$. Let $\lambda$ be any sense