A GENERALIZED CONVOLUTION FOR FINITE FOURIER TRANSFORMATIONS[1]

A. W. JACOBSON

1. Introduction. The finite sine transformation and the finite cosine transformation of $F(x)$ with respect to x are defined as follows:

$$S\{F(x)\} = \int_0^\pi F(x) \sin nx dx = f_s(n) \qquad (n = 1, 2, \cdots),$$

$$C\{F(x)\} = \int_0^\pi F(x) \cos nx dx = f_c(n) \qquad (n = 0, 1, 2, \cdots)$$

respectively. For example, the sine transforms of the first and second derivatives of $F(x)$ are $S\{F'(x)\} = -nC\{F(x)\}$ and $S\{F''(x)\} = -n^2 S\{F(x)\} + n[F(0) - (-1)^n F(\pi)]$.

If $F(x)$ in $(-2\pi, 2\pi)$ and $G(x)$ in $(-\pi, \pi)$ are bounded and integrable, then the function

$$F(x) * G(x) = \int_{-\pi}^{\pi} F(x - y) G(y) dy \tag{1}$$

is called the convolution of F and G on the interval $(-\pi, \pi)$.

If $F(x)$ and $G(x)$ are bounded and integrable on the interval $0 \leqq x \leqq \pi$, and if $F_1(x)$ is an odd periodic extension of $F(x)$ and $G_1(x)$ an odd extension of $G(x)$, then the product of the sine transforms of $F(x)$ and $G(x)$ can be written in terms of the transform of the convolution as follows:

$$S\{F_1(x)\} S\{G_1(x)\} = - 2^{-1} C\{F_1(x) * G_1(x)\}. \tag{2}$$

See [1, p. 274][2] and [2, p. 270].

2. The Fourier transformation of a generalized convolution. The purpose of this paper is to generalize the above results and to illustrate the use of the generalized convolution. This generalization consists, primarily, of extending the concept of the convolution to any

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[2] The numbers in brackets refer to the bibliography.